

# The cosmic distance scale

Introduction to Galaxies and Cosmology (AS3001)

- Laboratory exercise 1 -

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# 1 Introduction

The extragalactic distance scale is one of the fundamental topics of cosmology. This relates to all distances outside our galaxy, from nearby satellite galaxies through galaxy clusters all the way to the limit of our observable universe. Accurate measurements of extragalactic distances are needed to determine the age, size, structure and fate of the universe as well as estimating fundamental quantities like the amount of dark matter and the primordial densities of Hydrogen and Helium. In this laboratory exercise we will first use observations from the *Hubble Space Telescope* (HST) to calculate the distance to a galaxy called M100 in the nearby Virgo cluster of galaxies using so-called Cepheid variable stars. This will be done with a little help from a program written for this lab (called *Cepheid*) used to measure periods, magnitudes and to make plots of the Cepheids. We will then calculate distances to very far away galaxies using supernovas (these are very bright which means they can be seen over large distances). And finally, if all goes well, we will use this (together with velocities of the same galaxies) to calculate the expansion rate of the universe! But let us start with some preparatory exercises as a warm-up.

## 2 Preparatory exercises

Over the years a number of distance estimators have been found. Most methods rely on objects that can be used as so-called 'standard candles'. This is important since if we know how bright an object looks (apparent magnitude  $m$ ) and how bright it really is (absolute magnitude  $M$ ) we can calculate its distance. The trick is to use well known types of objects that behaves in a way that allows the absolute magnitude to be estimated.

### 2.1 The Magnitude Scale (exercise 1)

The astronomical magnitude scale is often regarded as quite strange by people who are not used to it and mistakes are therefore easily made when starting out using the scale. First of all it is reversed to common sense, with a brighter star having a lower magnitude. The faintest star seen in a dark sky using only ones eyes has a magnitude of  $m \approx 6$  while the full moon has  $m \approx -13$ . This also shows that the scale is not linear, in fact it is logarithmic and the difference between two magnitudes is defined as:

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2} \quad (1)$$

Where  $F_1$  and  $F_2$  denotes the corresponding fluxes. *Show that this equation can be rewritten as:*

$$\frac{F_1}{F_2} = \sqrt[5]{100^{m_2 - m_1}} \approx 2.512^{m_2 - m_1} \quad (2)$$

*If we have two stars,  $m_1 = 16.7$  and  $m_2 = 21.7$ , which star is brightest and how much brighter is it?*

All stars that we can see with our naked eyes are located in our galaxy, in fact they are very close to our position in the Milky way since we can only see stars brighter than  $m \approx 6$ . In sharp contrast to this, the stars used in the first part of this exercise belong to another galaxy (called M100) and appear in the sky as having  $m \approx 26$  but the Hubble Space Telescope (HST) is still able to detect them.

*How many times brighter would these HST stars have to be for us to be able to see them with our eyes?*

### 2.2 The mean magnitude (exercise 2)

Since the brightness of a variable star (like a Cepheid) varies in a repeatable way with some period (see Fig.1) we have to use its mean value for distance calculations. A simple approximation is to draw lines at the maximum and minimum values and use a line in between as the average. However, when we measure the flux in magnitudes we are using a logarithmic scale and this would only be a good approximation in a

linear scale. Remember, one magnitude is 2.512 times fainter in a linear scale, two magnitudes is  $2.512^2$  times fainter and so on...

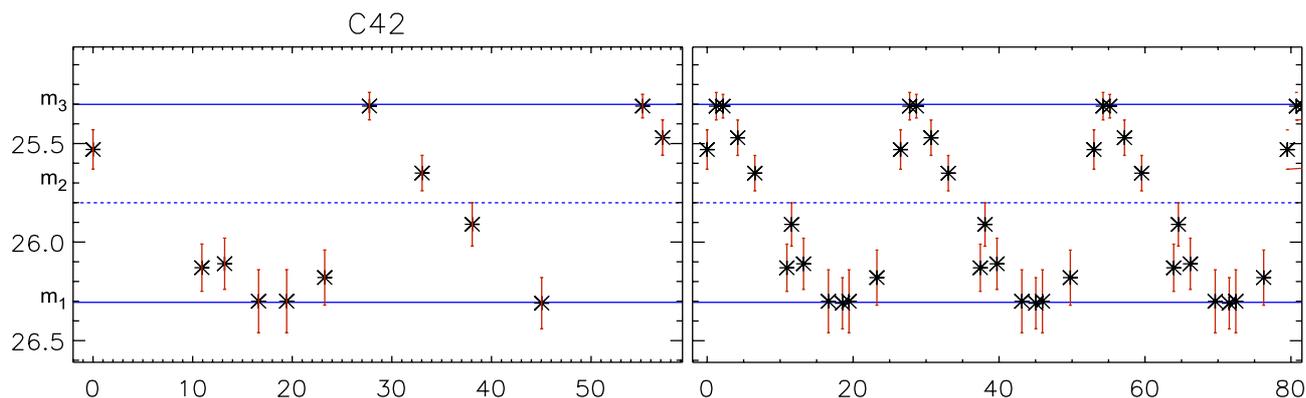


Figure 1: A Cepheid variable star with the magnitude plotted versus time (in days). Note the reverse scale on the y-axis because of the magnitude scale. In the right panel the same data is plotted repeatedly and with all points in every period, to better illustrate the variability.

We start by stating that in the linear scale we take the value between min and max flux as the mean:

$$F_3 - F_2 = F_2 - F_1 \quad (3)$$

In the logarithmic scale we instead use flux ratios: (derived in exercise 1)

$$F_2/F_1 = 2.512^{m_1 - m_2} \quad (4)$$

$$F_3/F_1 = 2.512^{m_1 - m_3} \quad (5)$$

Use equations 3–5 to find an expression that gives  $m_2(m_1, m_3)$ . Hint: Test your expression on the Cepheid in Fig.1 - if things have worked out, and I hope they have, you should get  $\langle m \rangle = m_2 \approx 25.69$

Note that  $m_2$  is *not* at the dotted line in Fig.1.

### 2.3 Distance modulus (exercise 3)

The apparent magnitude  $m$  (how bright something looks) has a relation to the absolute magnitude  $M$  (how bright something would look as seen from a standard distance of 10 pc) that can be used to calculate the distance to an object. We define the so-called distance modulus as:

$$\mu = m - M \quad (6)$$

Given the definition of magnitudes (Eq.1) and the inverse square law:

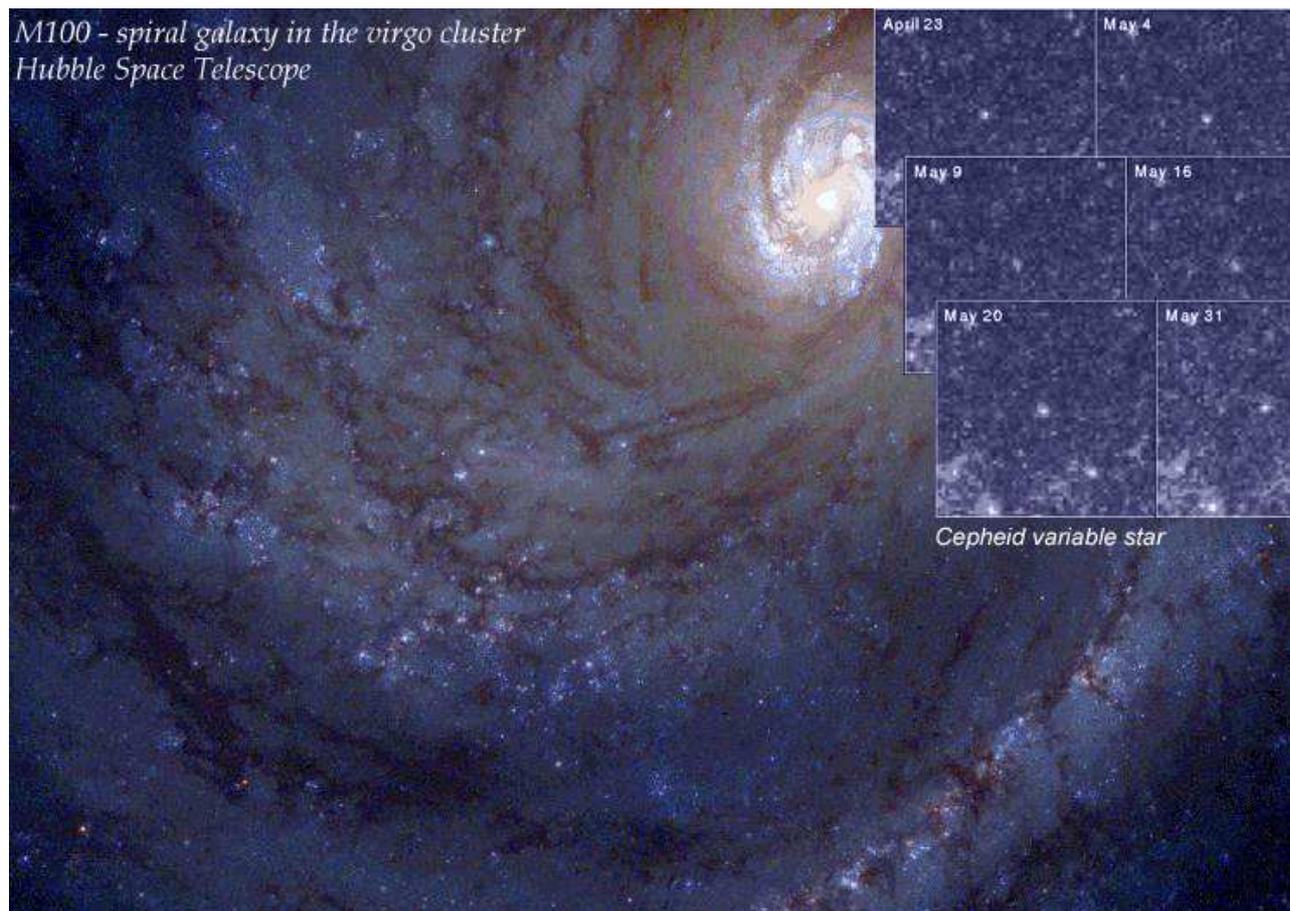
$$F = \frac{L}{4\pi d^2} \quad (7)$$

show that the distance  $d$  to an object (in parsecs) can be written as:

$$d = 10^{0.2(m - M + 5)} = 10^{0.2\mu + 1} \quad (8)$$

The Andromeda galaxy has  $m \approx 3.4$  and  $M \approx -21$ . What distance does this imply? convert your answer to lightyears, 1 parsec (pc) = 3.26 lightyears (ly).

### 3 Distance to the Virgo cluster of galaxies



One of the standard candles is a rare type of variable star called a Cepheid. These stars are very luminous, which is good since this makes it possible to observe them from a fairly large distance. Using the HST, individual Cepheids can be resolved as far away as the Virgo cluster of galaxies. Cepheids have periods from a few to about 70 days and this depends on their absolute magnitudes. In Table 1 HST observations of six Cepheids are listed. They were observed at 12 epochs (times) in order to find their periods. *Using this Table one can plot six diagrams, showing how the Cepheids vary in brightness over time, and estimate the period in each case.* For this we will use a program called “cepheid” on the students laptops, started by typing “idl\_6.0 -vm=cepheid.sav” in the proper folder (ask the lab assistants for details). The program contains the data in Table 1, and can be used to interactively find the period of each Cepheid by first selecting a Cepheid with the leftmost slider, then sliding the period slider or typing in a value. The upper plot shows the measured data (from the Table) and the lower plot a version with all measure-points included in all periods. When this curve becomes continuous, the period has been found for that Cepheid. For each solution, a plot can then be made to a postscript file and printed. Min and max apparent magnitudes can also be displayed for each Cepheid.

The absolute magnitude of Cepheids can be estimated using the following relation:

$$M_V \approx -2.76(\log(P) - 1) - 4.16 \quad (9)$$

where  $M_V$  denotes the absolute magnitude in the visual (V) band (standard filter used to observe about the same wavelengths as the human eye) and  $P$  is the period in days. *Use this relation to find absolute magnitudes for all your Cepheids.*

In order to calculate the distance modulus for the stars we need their apparent magnitudes  $m_V$  as well.

Table 1: V magnitudes of six Cepheids found in the galaxy M100 using the HST

Time [days]	C6	C19	C32	C37	C42	C50
00.00	25.51 ± 0.14	25.92 ± 0.18	26.07 ± 0.12	26.10 ± 0.17	25.53 ± 0.10	25.73 ± 0.09
10.93	24.78 ± 0.07	25.98 ± 0.18	26.57 ± 0.17	26.62 ± 0.24	26.13 ± 0.12	26.30 ± 0.16
13.21	24.82 ± 0.08	25.88 ± 0.14	26.40 ± 0.19	26.72 ± 0.24	26.11 ± 0.13	26.46 ± 0.16
16.62	24.87 ± 0.07	25.42 ± 0.11	25.59 ± 0.09	25.81 ± 0.12	26.30 ± 0.16	26.90 ± 0.24
19.44	25.00 ± 0.08	25.33 ± 0.09	25.59 ± 0.09	25.81 ± 0.12	26.30 ± 0.16	27.04 ± 0.29
23.26	25.11 ± 0.08	25.36 ± 0.10	25.86 ± 0.15	25.85 ± 0.16	26.18 ± 0.14	25.51 ± 0.11
27.75	25.47 ± 0.10	25.71 ± 0.14	26.01 ± 0.14	25.98 ± 0.16	25.31 ± 0.07	25.88 ± 0.11
33.04	25.55 ± 0.08	25.88 ± 0.16	26.40 ± 0.17	26.39 ± 0.22	25.65 ± 0.09	26.18 ± 0.13
38.07	25.59 ± 0.11	26.16 ± 0.20	26.74 ± 0.19	26.55 ± 0.20	25.91 ± 0.11	26.66 ± 0.22
45.04	25.60 ± 0.12	25.97 ± 0.16	26.76 ± 0.22	26.72 ± 0.25	26.31 ± 0.13	26.44 ± 0.14
55.17	24.89 ± 0.06	25.62 ± 0.14	25.82 ± 0.10	26.06 ± 0.15	25.31 ± 0.06	26.02 ± 0.11
57.18	24.83 ± 0.07	25.65 ± 0.12	25.86 ± 0.10	26.14 ± 0.19	25.47 ± 0.09	26.25 ± 0.13

Note - Cepheid C42 is already plotted in Fig.1

Use the relation you found in one of the preparatory exercises and the min/max magnitudes of each Cepheid to calculate the observed mean magnitudes. These have to be corrected for interstellar extinction. The light traveling to us from M100 is not just passing through the vacuum of space, some of it is absorbed by matter (dust) on its way. The amount of light lost varies between each star observed (the amount of intervening dust depends on the line of sight) *but on average we lose  $A_V \approx 0.3$  magnitudes due to extinction* (denoted A, and the subscript V shows that it is in the green filter) *that we must subtract to correct for this* (remember: lower magnitudes mean brighter objects).

Now, using these six distance modulus calculate the distance to each of the Cepheids and finally, estimate the distance to M100 (with an uncertainty estimate) from the mean and standard deviation of the six distances.

In order to make an estimate of the expansion rate of the universe (the Hubble constant) we need to know how fast galaxies at the distance of M100 are moving away from us. However, since M100 is part of a large cluster of galaxies, this cannot be done by simply making a velocity measurement on this galaxy alone. The galaxies in a cluster move around the centre of the cluster, just like the planets in our solar system around the Sun - what we need to find out is the radial velocity of the whole Virgo cluster. In Fig.4 spectra of ten galaxies in Virgo are shown, zoomed in at a region around the H $\alpha$  transition (atomic Hydrogen). The rest wavelength (lab frame) of this transition is located at 6562.8 Å and is marked in the figure with a red dashed line. From the definition of redshift (z) we can use the Doppler shift of this line to calculate the velocity of each galaxy in the sample according to:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c} \quad (10)$$

Do this for the ten galaxies and calculate the velocity of the Virgo cluster from the mean velocity. Now we have a distance and the expansion velocity at that distance. Use these to estimate the Hubble constant  $H_0$  (remember that the units of this constant is km/s/Mpc). Compare this to the literature value. Remember, this is a very rough estimate and we should really also use galaxies at much further distances to get reliable calculations (more about that in the next part).

## 4 Expansion rate of the Universe

Now we have learnt how to use Cepheid variables to measure distances. But what if we want to measure distances much further away than the Virgo cluster (and we do!). We would need a type of object that is much brighter than a Cepheid that also has a known absolute magnitude so that we know how much light is emitted. Type Ia supernovae<sup>1</sup> are very bright, in fact about 14 magnitudes brighter than Cepheids. These so-called secondary standard candles must be calibrated to the first step in our cosmic distance ladder (our Cepheids).

For this we need to find a recent supernova of this type in one of the Virgo cluster galaxies, preferably in M100 where our Cepheids are. However, given that supernovae are rare this is a best case scenario. Believe it or not, but in early February 2006 Shoji Suzuki (Japan) and M. Migliardi (Italy) independently discovered a supernova of type Ia in M100 (see Fig.2) which peaked a few weeks later at  $m_B = 15.36$  (B = Blue filter), remember that our Cepheids were at about 25-26 magnitude so this is much brighter - and just what we need.

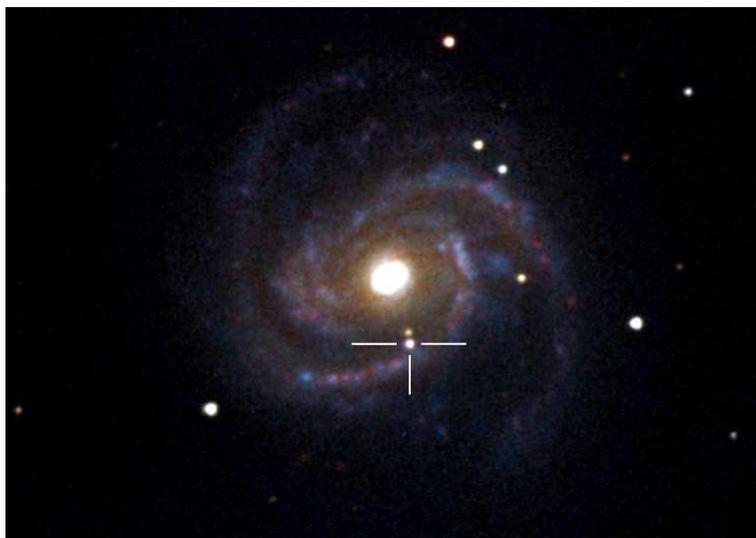


Figure 2: Discovery image of SN2006X in M100.

Earlier it was mentioned that all SN Ia ought to have the same absolute magnitude because the white dwarf star of the system explodes when it reaches a certain mass (which applies everywhere in the universe). This is however not exactly true, different white dwarfs have different atmospheric compositions and this in turn affects both how fast the supernovae fade and how bright they become, see Fig.3.

*The total extinction (in the blue band) towards SN2006X has been found to be  $A_B \approx 3.40$  magnitudes due to dust between us and the supernovae (mostly in the host galaxy) - we have to correct our apparent magnitude  $m_B$  for this. Use Eq.8 to find the peak absolute magnitude  $M_B$  of SN2006X. For simplicity, we will assume that SN2006X is typical and that all supernovae Ia have this luminosity (even though Fig.3 shows otherwise). This is certainly a good enough assumption for an exercise, however, using many more supernovae than just one to calculate the *mean* peak luminosity would of course be much more reliable. The principle is however the same and the assumption saves us a lot of calculations.*

Table 2 contains redshifts ( $z$ ) from spectroscopy and photometry (apparent magnitudes  $m_R$ , i.e. red filter) of seven supernovae. Column 4 gives the extinction ( $A_R$ ) due to dust in our own galaxy. Because the cosmological redshift varies with distance, the spectra of these SN will be redshifted by different amounts

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<sup>1</sup>A Type Ia Supernova (SN) is a binary system where mass is transferred from a companion to a white dwarf. When a certain mass limit is reached the white dwarf explodes. Since this mass limit (called the Chandrasekhar limit) is universal, these supernovae are believed to have about the same absolute magnitudes.

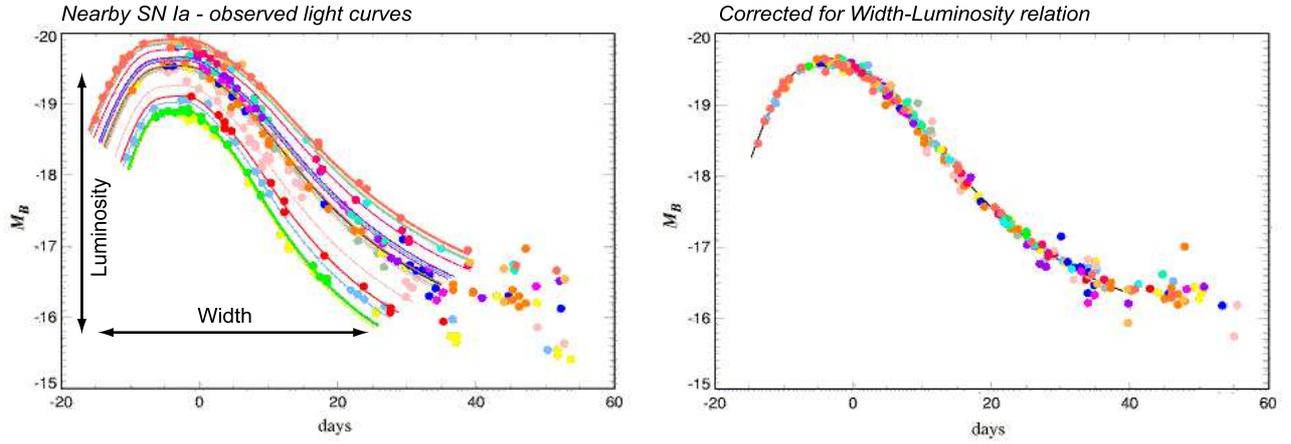


Figure 3: Supernova Ia vary somewhat in peak luminosity because they have different compositions. A linear relation has however been found between the peak absolute magnitude and the width of the light curve (at  $z=0$ ). Short lived supernovae are fainter - but when the relation is taken into account we get a standard curve (right panel).

Table 2: Supernovae table

Name	$z$	$m_R$	$A_R$	$K_{RB}$	$\Delta_{corr}$
SN 1992bi	0.458	22.01	0.003	0.70	0.55
SN 1994H	0.374	21.38	0.039	0.58	0.16
SN 1994al	0.420	22.42	0.228	0.65	-0.06
SN 1994F	0.354	22.06	0.010	0.56	-0.81
SN 1994am	0.372	21.73	0.039	0.58	-0.25
SN 1994G	0.425	21.65	0.000	0.66	0.05
SN 1994an	0.378	22.02	0.132	0.59	-0.47

(remember:  $\Delta\lambda = z\lambda$  for a given wavelength). This means that spectra are redshifted and stretched with distance. To be able to compare the SN with each other we must therefore correct for this. For a SN at  $z \approx 0.4$  the light sent out in the blue filter wavelength range would actually arrive in the red filter but for other redshifts we get a mismatch, correcting for this and the stretching is done by a 'K-correction' which is given in Column 5 (the RB subscript denotes that we are using R and B filters). And in the final column we have a correction for the Width-Luminosity relation shown in Fig.3 but applied on the apparent magnitude  $m_B$ . There is also a stretching of light curves with increased  $z$ , but this has been accounted for in the corrections.

*All this means that we can calculate 'effective' apparent blue magnitudes for our sample:*

$$m_{B,eff} = m_R - A_R + K_{RB} + \Delta_{corr} \quad (11)$$

And since we already have calculated  $M_B$  using SN2006X in M100, we can now calculate the luminosity distance to all these supernovae (see Eq.8) and also their velocities (for simplicity we use the non-relativistic approximation  $v = cz$ , see Eq.10). After you have done this, make a Hubble diagram that includes both the Virgo cluster from the first part of the exercise and these SN. Plot the velocity versus distance and estimate the Hubble constant  $H_0$  by fitting a line in this diagram. Note that the  $v \approx cz$  approximation breaks down for high  $z$  objects (otherwise we would have  $v > c$  for  $z > 1$ ) so it might be a good idea to skip some of the highest  $z$  points in the fit.

*Some questions to be answered:*

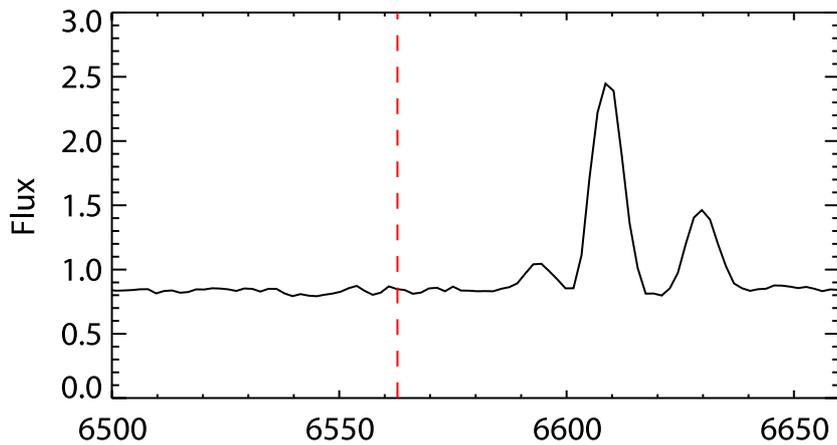
Have we taken into account the extinction of the host galaxies of these Supernovae? If not, how would this affect our Hubble diagram?

How does the deceleration parameter  $q_0$  change our Hubble diagram?

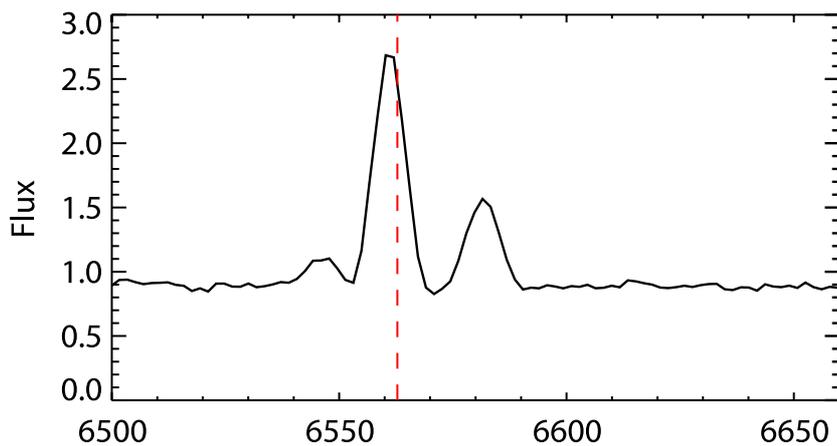
Why are SN type Ia expected to be a good standard candle?

The light curve of distant (high  $z$ ) SN are stretched out compared to one at  $z=0$ , what is the reason for this?

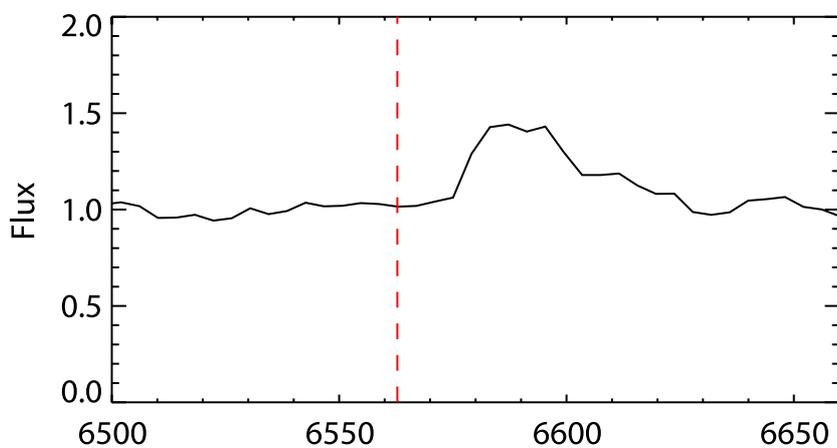
Make a morphological classification of the Virgo cluster galaxies for which you have determined the redshifts and distances. Why are there no elliptical galaxies for which a distance based on Cepheid stars can be obtained?



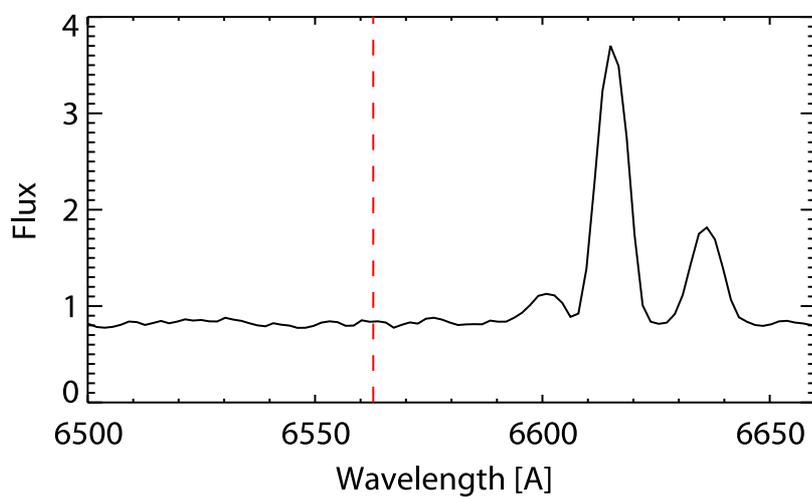
NGC 4189



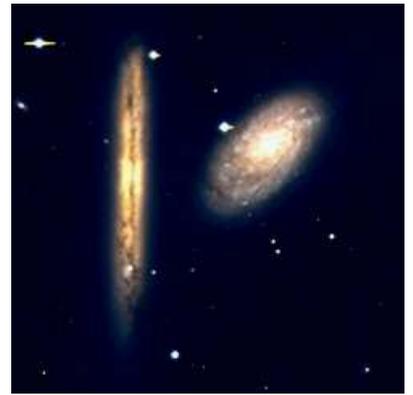
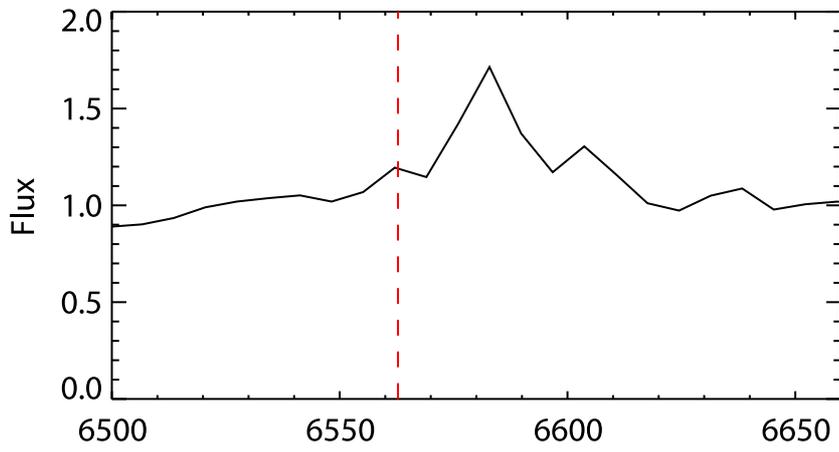
NGC 4212



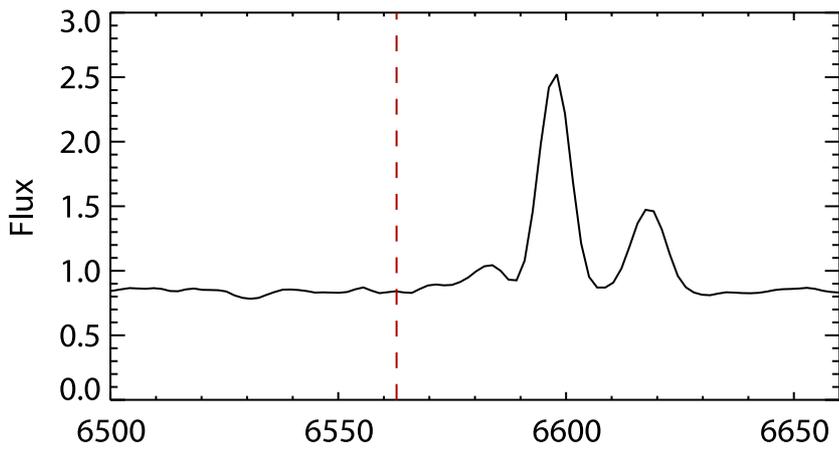
NGC 4237



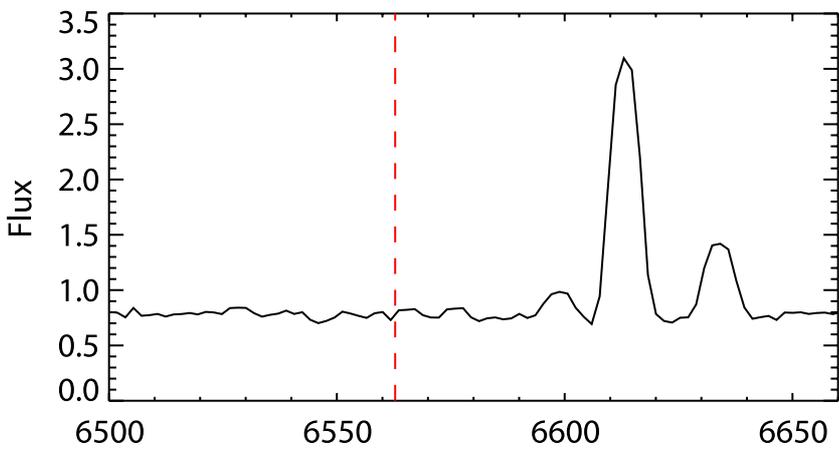
NGC 4254



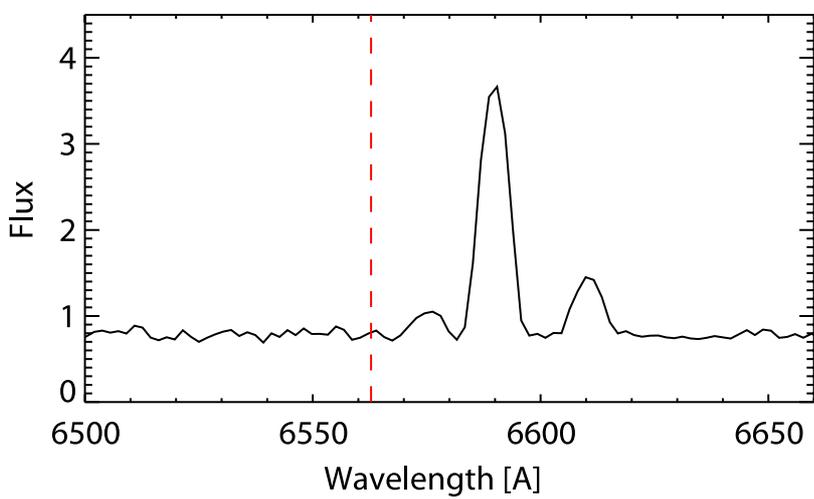
NGC 4298



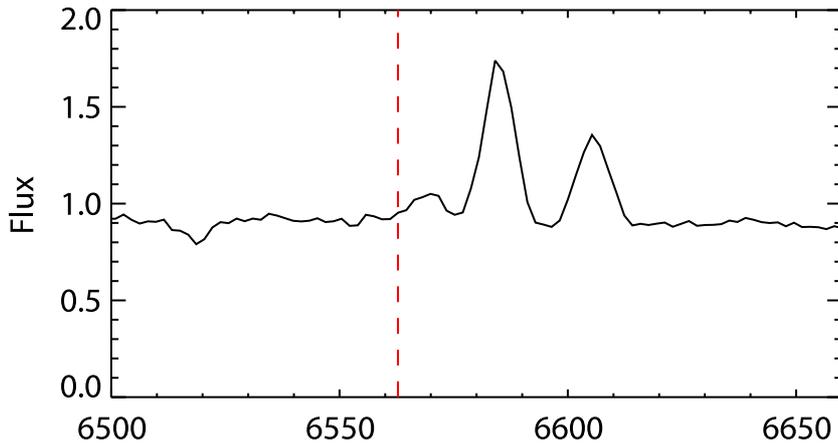
NGC 4321 (M 100)



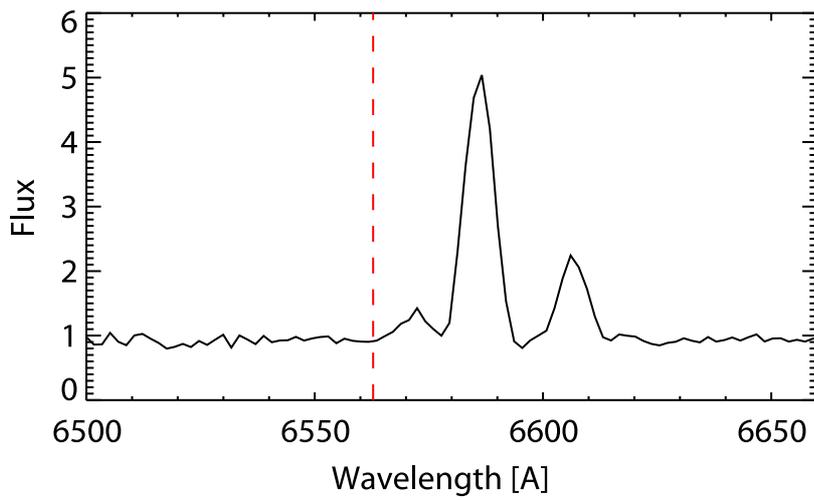
NGC 4351



NGC 4519



NGC 4639



NGC 4654

Figure 4: Optical spectra of 10 galaxies in the Virgo cluster. These figures only show a small region around the H $\alpha$  line (rest wavelength 6562.8 Å, marked with a red dashed line) - making it possible to measure the radial velocity of each galaxy (because of the doppler shift of light).