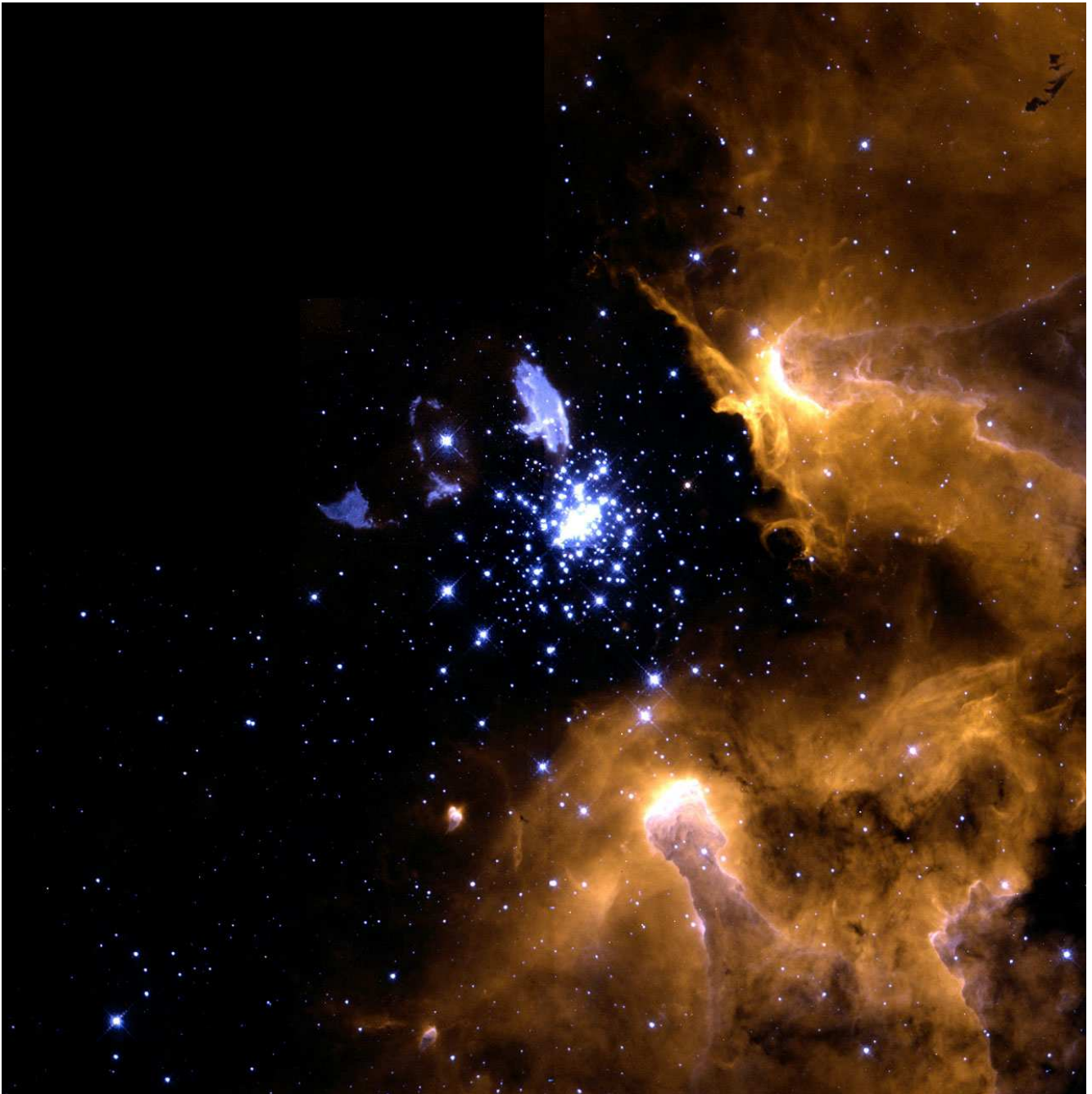


Modelling stars on the Zero-Age Main Sequence

Laboratory exercise - Stellar Structure & Evolution

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1 Introduction

In this exercise you will use a computer program to generate models of stars on the zero-age main sequence (ZAMS) in order to examine and compare their properties. The program needs initial guesses of model parameters as input, it is therefore important that the preparatory exercise has been done before starting the computer lab. The modelling program is called `zams.f` and comes from the book *Stellar Interiors* by C. J. Hansen and S. D. Kawaler.

It is worth reading chapter 2.2 of this book, “Single Stars On and Near the Main Sequence,” to get an overview for this project. You can also read a bit more about the computer code itself in Appendix C. Finally, if you are interested in more details of stellar modelling, chapter 7 of the book contains information about the numerical methods used to produce such models.

This version of the lab. uses a graphical interface (programmed in IDL by M. Gålfalk) in which you will be setting up model parameters, running the models and printing to postscript files.

2 Preparatory exercise

One of aims of this lab is to learn how to calculate reasonable guesses about stellar parameters for a given mass. These estimates are also required as input parameters when making the models. Because of limitations in the modelling method and in order to check that the estimates are roughly accurate, the modelling program has been written so that a solution must be found within a fair number (50) of iterations.

We will start by considering the following set of equations of stellar structure for a ZAMS star with mass $M > 1 M_{\odot}$ where nuclear energy is released mainly by hydrogen burning via the CNO cycle and the energy flux is dominated by radiative transport:

$$\begin{aligned}\frac{dr}{dm} &= \frac{1}{4\pi r^2 \rho} \\ \frac{dP}{dm} &= -\frac{Gm}{4\pi r^4} \\ P &= \frac{\rho}{\mu} \mathcal{R}T \\ \frac{dl}{dm} &\approx \epsilon_0 \rho T^{15} \\ \frac{dT}{dm} &= -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{l}{(4\pi r^2)^2}\end{aligned}$$

Assume a homogenous composition throughout the star ($\mu = \text{constant}$) and that Thomson scattering is the main contributor to the opacity (i.e. $\kappa = \kappa_e = \text{constant}$).

In order to avoid having to solve this system of equations for each model we are making, we will simplify things by using homology - i.e. assume that the stellar parameters (r , P , ρ , T and l) vary in the same way with relative mass radius for all stars with the addition of scaling factors (R_S , P_C , ρ_C , T_C and L).

By introducing a dimensionless relative mass radius $x = m/M$ we can reformulate our parameters:

$$\begin{aligned}r &= f_1(x) R_S \\ P &= f_2(x) P_C \\ \rho &= f_3(x) \rho_C \\ T &= f_4(x) T_C \\ l &= f_5(x) L\end{aligned}$$

The functions f_1 – f_5 are thus the same for all stars independent of total mass, while the scaling factors are the estimates we are looking for.

- (a) By means of dimensional analysis derive how density, pressure, luminosity and temperature (roughly) scale with each other, mass M and radius R of the star. For those of you unfamiliar with dimensional analysis, this is done by substituting the reformulated parameters into our set of stellar structure equations and rearranging everything dimensionless (like f_1 , df_2/dx , π) to one side and the rest to the other side of each equation. We then know that the dimensional side of each equation (expressions including e.g. G , M , R_S , ρ_S) must equal a constant. (e.g. from the first equation you have $dm/dr = 4\pi r^2 \rho \Rightarrow M/R_S \sim R_S^2 \rho$, etc. . .)
- (b) From the results of the dimensional analysis you can actually find that the luminosity $L = l(R_S)$ of the star scales approximately as a power of the mass M , $L \sim M^\nu$: what is the value of the exponent ν ?
- (c) How does the radius R_S of the star scale with the mass M ? And what about the central temperature T_c , central pressure P_C and central density ρ_C ?
- (d) According to your scaling relations, which star has the largest radius: a low or high mass star? What about central density and central pressure?
- (e) With the help of the scaling laws you derived, find how surface gravity g and effective temperature T_{eff} scale with M .

3 The program

The program that we will use to make our ZAMS models is simply called *stellarstruc*, written by M.Gålfalk in the IDL language. The calculations are made in a Fortran program called *zams*, however, as this is a terminal program without a graphical interface or means for printing, *stellarstruc* was written with the aim to simplify the model making and printing of results.

Ask the lab assistant for details on where the program is located on the student laptops. To start *stellarstruc* simply open up a terminal window (such as the *bash* shell from the Debian/Apps/Shells menu). Go to the appropriate folder (using the *cd* command) and type the following two commands:

```
PATH=$PATH: .
idl -vm=stellarstruc.sav
```

Instructions for how the program works is found in the next section.

It may be necessary to also compile the Fortran code before launching *stellarstruc* if this has not been done previously (if there is a *zams.f* but no *zams* file). This is simply done by typing:

```
f77 -o zams zams.f
```

4 Making the models

The interface of the program is shown in Figure 1. The numbers given within parenthesis in the text below refers to those in the figure.

Most of the **units** in the program are cgs-units. Thus, we have temperature T [K], pressure P [dyn/cm²], radius R [cm], luminosity L [erg/s], density ρ [g/cm³], opacity κ [cm²/g] and energy production rate ϵ [erg/s/g].

In the plots used for printing, the logarithmic versions of these are plotted. The exceptions to cgs-units are the total mass [M_\odot] and luminosity [L_\odot] in the input parameters and the luminosity iteration plot. In all plots, the x-axis is the relative mass radius, M_r/M_{tot} with M_r being the mass out to a certain radius and M_{tot} representing the total mass (thus M_r/M_{tot} ranges from 0 at the core to 1 at the stellar surface).

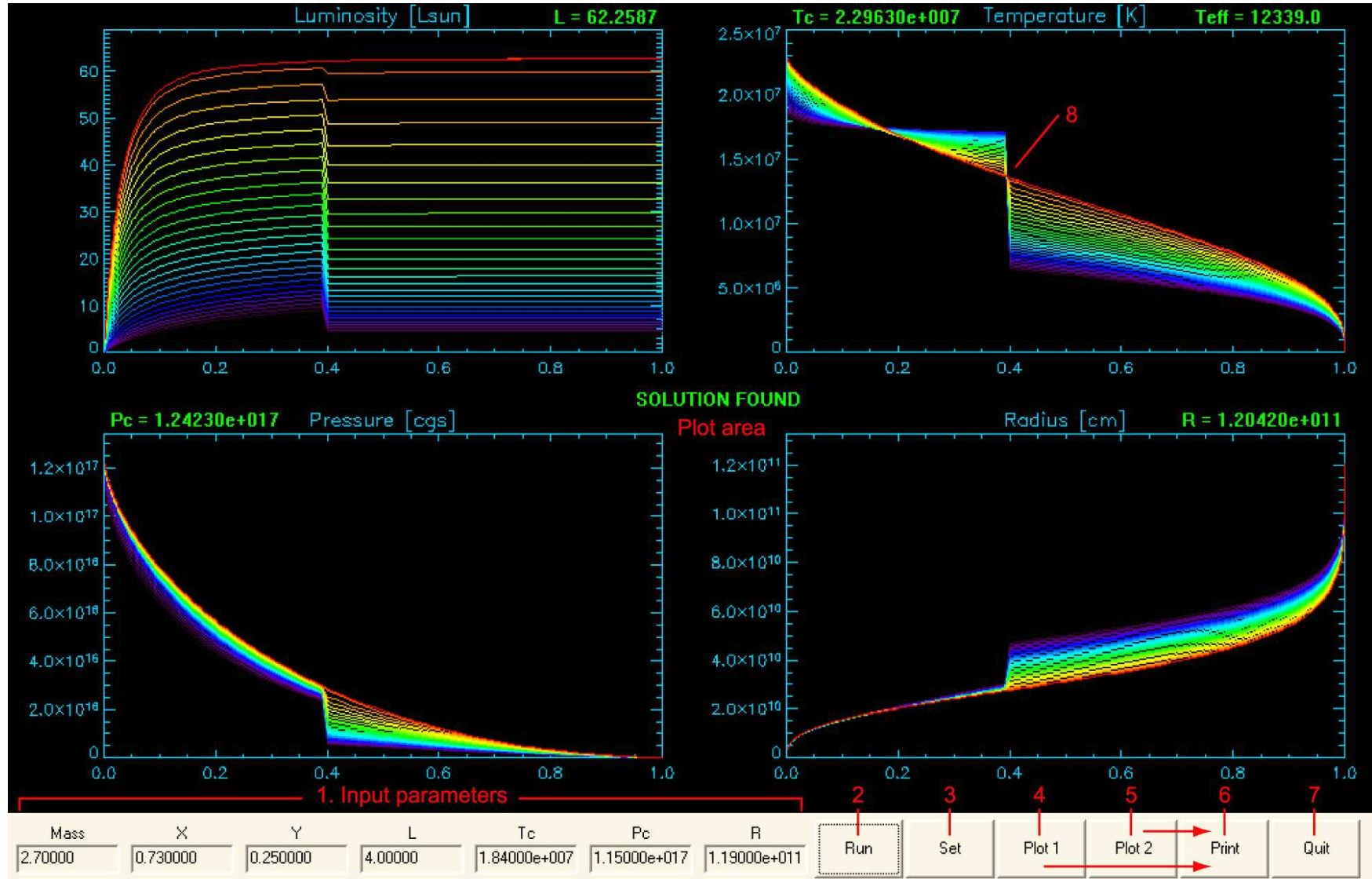


Figure 1: The graphical interface of *stellarstruc*. Input parameters are entered into the boxes at (1), a model iteration is launched by pressing button (2). If a solution is found this is shown at the centre of the screen and important parameters are shown in green. The Set button (3) copies the most recent solution to the input parameters. Buttons (4) and (5) makes plots of the results needed for the exercises, after which you can press button (6) to save the current plot to a postscript file for printing.

- For each model we must enter some input parameters (1) - the mass of the star ($M [M_{\odot}]$), the chemical composition (X and Y , the hydrogen and helium mass fraction) and the initial guesses of four parameters (total luminosity $L [L_{\odot}]$, central temperature T_C , central pressure P_C and the radius of the star R_S).
- After running a model (2) the program will iterate the initial guesses, drawing a curve in each panel of the plotting area for each iteration. The way this works is that the program integrates the differential equations of the model both from the centre and from the surface. If a model is discontinuous at the fitting point (8), somewhere in between the core and the surface, e.g. a jump in the temperature curve, the program will adjust the guesses and make a new iteration. This goes on until a continuous solution has been found. There is however (on purpose) a limit of 50 iterations for this to happen since making reasonable *initial guesses* (based on the preparatory exercise) is part of the lab. The program may also abort the iterations at once if the initial guesses are way off. By using your scaling relations (hopefully) found in the preparatory exercise you should not be that far off except perhaps for very high mass stars (since guesses at higher mass made from scaled $1 M_{\odot}$ parameters also scales the uncertainties).
- The set button (3) can be very useful in the guess work. It copies the solution parameters from the most recent successful model. Thus, if everything goes wrong you can use this to get back to the latest working model. Also, if you copy the solution from a model with the set button, these numbers can be used as input parameters for another model with similar mass. E.g. the solution from a successful $5.0 M_{\odot}$ model can be copied to the input parameters with the set button and used as guesses for a $6.0 M_{\odot}$ model, after changing only the mass parameter.
- For any successful model where a solution has been found, important parameters are shown in green next to the plots. Writing these down on a piece of paper is necessary for any model that will be used in the exercises. Also, eight solution plots can be made using buttons (4) and (5) - and written to postscript files for printing using button (6). Only the plots displayed on the screen (using buttons 4 or 5) will be saved to a file with button (6). The files are named using the total mass and metallicity ($Z=1-X-Y$) of the model, preceded by *plot1* or *plot2* depending on which plot screen is displayed. E.g. a model with $M=2.70$, $X=0.73$ and $Y=0.25$ will be saved as *plot1_M2.70_Z0.020.ps* if button (4) has been clicked to create the plots and (6) to print these to a file.
- By pressing anywhere on the plot area the solution is cleaned up and only the final iteration (the red curve) is shown. This can also be done to get back to the main window from the *plot1* or *plot2* modes.
- Button (7) is used to quit the program.

Note that numbers such as 6.93×10^{10} can be entered in exponential notation (6.93e10).

Plots saved to a postscript file can be checked out using a program such as *ghostview* (by typing *gv filename.ps* in a terminal window). If you are happy with a plot it can easily be printed from *ghostview* by selecting the *File/Print document* option or (directly from a terminal window) by typing a printer command *lp -d printername filename*.

5 The laboratory exercises & questions

The idea of this lab is to get an overview of the properties of main-sequence stars, modeled just as they have entered the MS (homogenous chemical composition since fusion has just started). Here are some questions that should be investigated and answered in your report:

1. Read the description in chapter 2.2 of Hansen and Kawaler about the stellar modelling code. What approximations have been made and what would be needed to get more realistic models? What is the range of applicability of the models, i.e., in what range are the models reasonably realistic and when should you be a bit skeptical of the model output? Also, why is the surface composition (X and Y) of our current Sun a good assumption for the composition (X and Y) throughout a $1 M_{\odot}$ ZAMS star?
2. Use the scaling relations you found in the preparatory exercise to make solar composition ZAMS models ranging from 0.8 to about $70 M_{\odot}$ (e.g. $1, 2, 4, 8, 16, 32, 64 M_{\odot}$). For the $1 M_{\odot}$ model we can use the

parameters of the Sun today ($X = 0.73$, $Y = 0.25$, $P_C = 2.477 \times 10^{17}$, $T_C = 1.58 \times 10^7$, $R_S = 6.96 \times 10^{10}$) and scale these to make initial guesses at all other masses. Remember to make a $0.8 M_\odot$ model as well!

3. Choose three stars of different masses and investigate important quantities as functions of relative mass radius. Plots can be made using the *plot1* and *plot2* buttons in the program and printed from the postscript files.
4. Examine the different temperature gradients and explain how to identify the convective zones in your models. Use plots to get an overview of the different extent of convective zones in models of different masses, and discuss the reasons for those differences. In the report you should include the $0.8 M_\odot$ model, a high mass and an intermediate mass model. The plot of interest for this exercise is the lower right panel using the *plot2* button. The solid curve represents ∇_{ad} and the dashed curve ∇_{rad} , where $\nabla = d \ln T / d \ln P$. This plot can be used to determine the convective and radiative zones in the three models.
5. How do the central temperature, central density, effective temperature and energy generation rate change according to the mass of the star? Interpret these results. Do the first three parameters scale with mass as you found in your scaling relations?
6. Plot the total mass-radius and total mass-luminosity relations using your model results. How well do these agree with the exponents you found in your scaling relations?
7. Now study the effect of metallicity ($Z=1-X-Y$) by comparing models with two very different chemical compositions, one series of very low-metallicity (Z_{low}) models and one series of very high-metallicity (Z_{high}) stars. Use the same masses and initial guesses as for your solar composition models, but remember to change the chemical composition parameters (X and Y) to get the metallicity Z you want. Use two reasonable, but very different metallicities (where would we find such very low and high metallicity stars in reality?). For a given mass, which of the two chemical compositions has the largest central temperature? The largest central density? The largest energy generation rate at the center? Explain why this may be so.
8. Use your data to plot the zero-age main sequence (ZAMS) in a traditional $\log \mathcal{L}$ vs. $\log T_{eff}$ Hertzsprung-Russell diagram. How does the ZAMS differ for stars of different chemical compositions? How do you account for the differences in effective temperature and luminosity between two models of the same mass but different chemical composition?

The Fortran code used in this exercise was developed by C. J. Hansen and S. D. Kawaler, in *Stellar Interiors: Physical Principles, Structure and Evolution*, Springer-Verlag, 1994. This computer exercise (version 2008 by Magnus Galfalk) was partly based on a lab. at Uppsala observatory (version 2006 updated by Ola Karlsson), version 2005 by Remo Collet, based on the one by Nils Ryde (1997) and Michelle Mizuno-Wiedner (2003).