Radiation

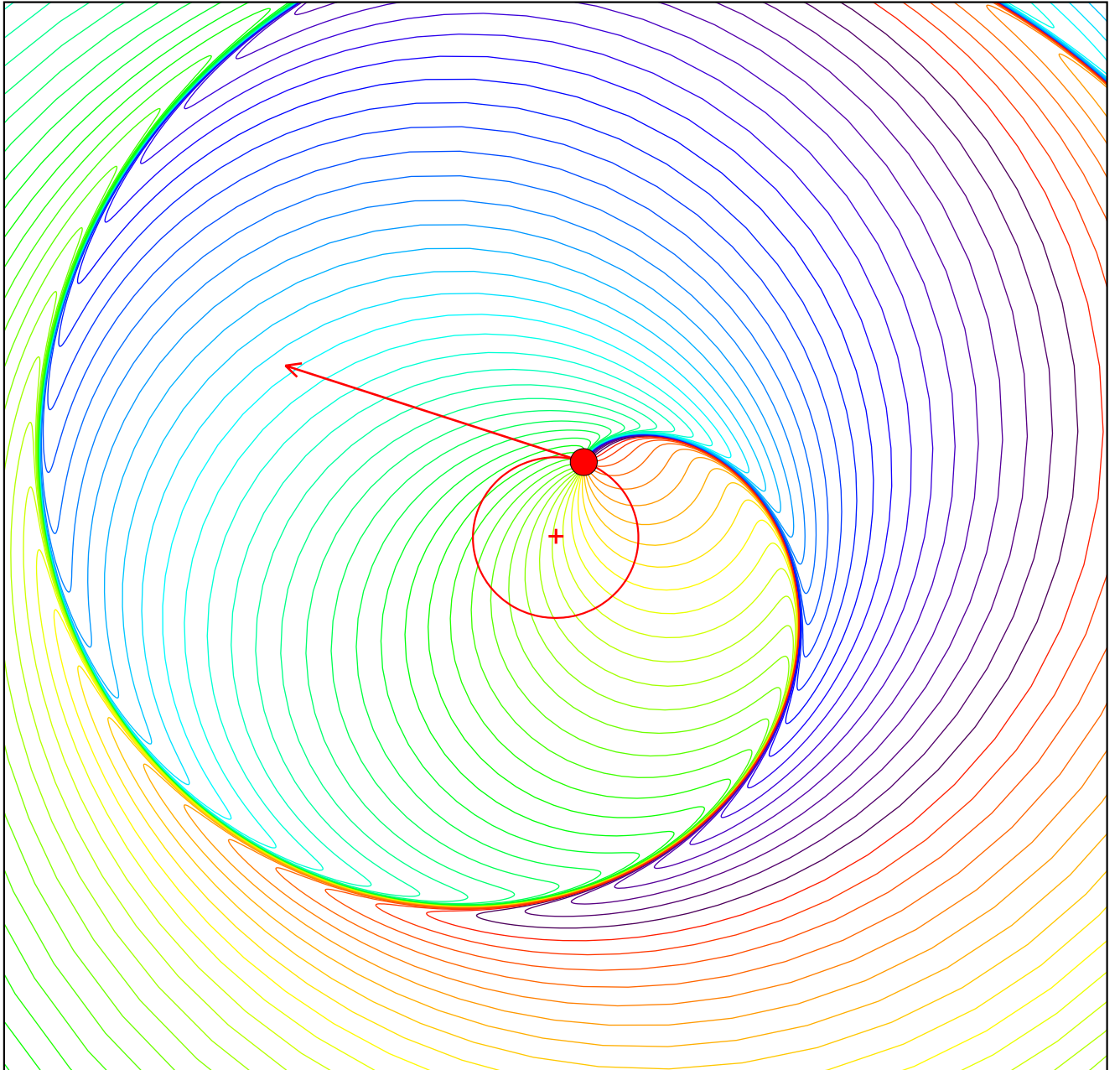## Laboratory exercise - Astrophysical Radiation Processes

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1   Introduction

The electric (and magnetic) field pattern from a single charged particle can get very complicated at relativistic velocities, and diverge a lot from the non-relativistic case with lines radially outward. In this lab we will use a program (*Radiation 1.0*) to investigate the behaviour of the electric field from a charged particle at different velocities in circular motion, linear oscillation and instant change of velocity. Clasically, electromagnetic radiation is emitted from an accelerated charged particle. These motions are reminiscent of several physical processes that cause radiation: braking radiation (bremsstrahlung), dipole radiation, cyclotron and synchrotron radiation.
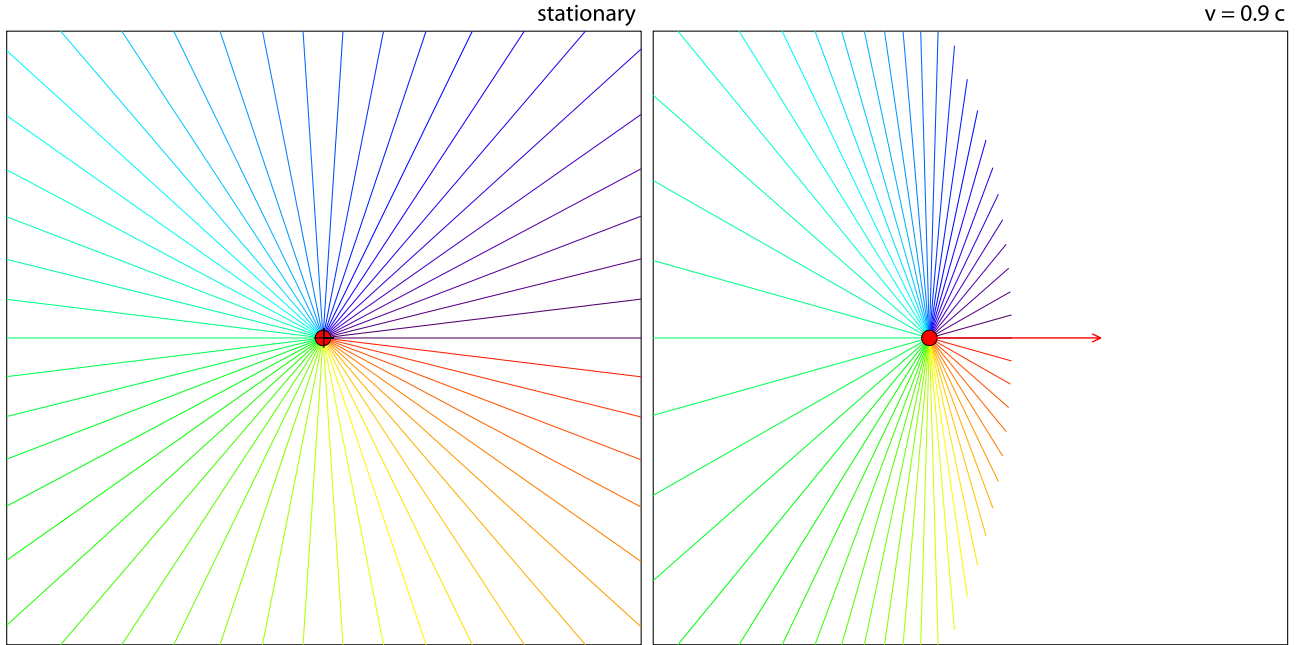

Figure 1: A charged particle at rest (left panel) and moving at $v = 0.9c$ (right panel) seen in the laboratory frame. In the stationary case we have uniformly distributed field lines, spherically outward. In the relativistic case, however, a uniform distribution of field lines (in the frame of the particle) is transformed into a nonuniform distribution of straight field lines - in the shape of an oblate spheroid pattern orthogonal to the direction of motion.

2   Some theory

As shown in Fig. 1, a charged particle at rest or uniformly moving at non-relativistic velocities (in the laboratory frame) has a simple electric field with field lines radially outward from the current position of the charge. In the frame of the particle the field lines are always uniformly distributed, but as seen in the lab frame, we get the following transformation law

$$tan(\theta') = \gamma tan(\theta) \tag{1}$$

where θ is the angle of a field line with the direction of motion in the charge's rest frame and θ' is the same angle as viewed from the laboratory frame and γ and β are defined as

$$\gamma = \left(1 - (v/c)^2\right)^{-\frac{1}{2}} = \left(1 - \beta^2\right)^{-\frac{1}{2}} \tag{2}$$
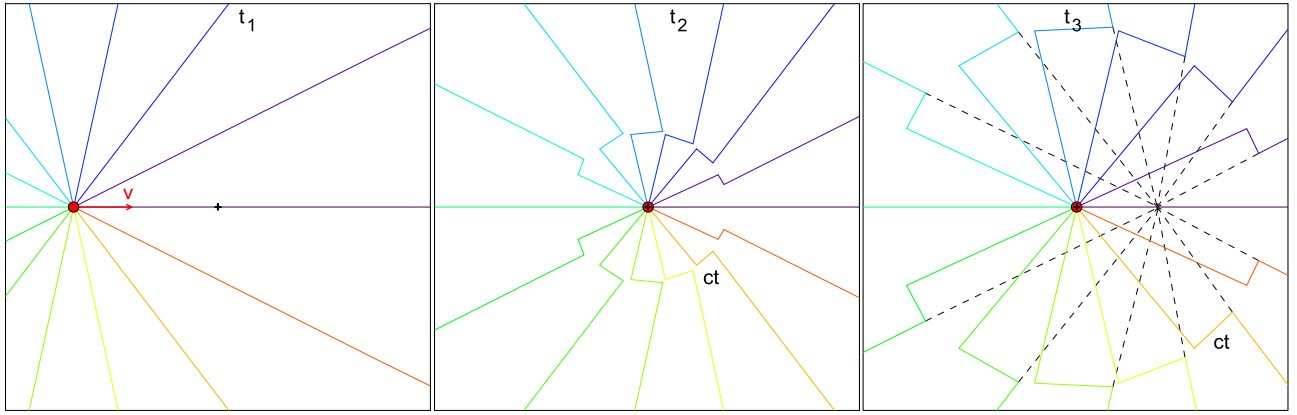
$$\beta = \frac{v}{c} \tag{3}$$

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Figure 2: A charged particle moves at velocity *v* and stops instantaneously. In the left panel we see that the particle has an electric field similar to a particle at rest, with field lines radially outward (at low speeds). At a time t_2 (middle panel) the particle has stopped and since information cannot travel faster than the speed of light this fact is unknown to the field outside a sphere with radius $r = ct$ where t is the time since the particle stopped. At a later time t_3 (right panel) this *radius of information* has reached further out. Tracing the electric field lines inwards from $r > ct$ (dashed lines) we see that they converge where the particle would have been had it not stopped.

All particle paths in this lab are approximated by very small linear segments building up a curve (a new segment for each step in the animation). Each time the particle changes velocity (speed and/or direction) the field lines change. This is called an *event*, however, since information cannot travel faster than the speed of light there is no way for the field lines outside a radius of $r = ct$ to know about the change in velocity. Thus, as can be seen in Fig. 2, the field lines further out act as if the event had not taken place, converging at a point where the charge *would have been* without the change in velocity.

Since electric charge is a Lorentz invariant, the number of field lines has to be conserved across the expanding surface $r = ct$. Therefore we get a spherical boundary region, forming a radiation pulse transverse to the velocity of the growing sphere. We can compute the position of the field lines by obtaining the angle of the intersection of each line with the growing sphere (this angle in independent of time). After some math exercise for general two-dimensional motion we obtain

$$X(t) = x(t - t') + ct'cos(\theta') \tag{4}$$
$$Y(t) = y(t - t') + ct'sin(\theta') \tag{5}$$

and

$$cos(\theta') = \frac{cos\alpha(cos(\theta - \alpha) + \beta)}{1 + \beta cos(\theta - \alpha)} - \frac{sin\alpha sin(\theta - \alpha)}{\gamma(1 + \beta cos(\theta - \alpha))} \tag{6}$$

$$sin(\theta') = \frac{sin\alpha(cos(\theta - \alpha) + \beta)}{1 + \beta cos(\theta - \alpha)} + \frac{cos\alpha sin(\theta - \alpha)}{\gamma(1 + \beta cos(\theta - \alpha))} \tag{7}$$

where x(t-t'), y(t-t') is the position the charge had at a time t' in the past from the current time t (t-t' thus corresponds to the time of the event that caused the growing sphere). The angle α is here defined as the angle between the velocity vector of the charge and the positive x axis at the time of the event. The program in this lab (see next section) uses equations 4–7 with a new *event* for each calculated step in the animation.

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3 The program

The program we are going to use in this lab is called *Radiation 1.0*[1]. It is written in IDL and is easy to run using most platforms (Windows, Unix, Solaris, Mac) as long as IDL virtual machine is installed. To run the program simply type "idl -vm=radiation.sav" in the proper folder (ask the lab assistants for details). When *Radiation* is launched it shows a menu where one can select between four types of paths for the charge, each case with its own interface. These interfaces are outlined in detail for each case: Instantaneous velocity change (Fig.3), Oscillating charge (Fig.4), Circular motion (Fig.5) and Fun with splines (Fig.6). While the first three cases are part of the lab, the spline-curve case is optional, however, it is a fun and illustrative way to make your own custom path and play around with electric fields at relativistic velocities.

The program works by moving the charge in very small steps, treating each new step (frame) in the animation as an event governed by equations 4–7. It thus has to keep track of hundreds of different events at the same time throughout the animation. The calculations are done in real time (not pre-recorded) meaning that they can be made for any type of motion curve and for as long as needed, however, to speed up the drawing, field lines far out from the charge (often outside the screen) are dropped from the calculations.

For all motion paths, the number of electric field lines can be set and is simultaneously displayed as a preview using a test charge in the lower left corner of the window (item 1 in the interface Figures). Since the speed of computers vary a lot, both regarding calculations and drawing, there are two parameters that can be changed to speed up or slow down the animation. The *Delay* slider (usually set to 0 for no delay) can be increased if necessary to increase the time each step in the animation is shown (for example if very few field lines are used and the animation displays too fast). A fast computer can easily handle the maximum number of field lines (100) and display the animation smoothly.

When the settings are done a simulation is started by pressing the *Run simulation* button. To get back to the settings after an animation is done just click the left mouse button anywhere in the window. Any animation can be plotted to a postscript file for easy printing and for including in the lab report. The result is 6 frames (3×2 plots on a landscape A4 page) uniformly spaced in time, illustrating the animation. For cyclic animations (linear oscillation and circular path) the program automatically prints one period even if several periods are set in the animation's *Duration* setting.

4 The exercises

In the report, the exercises in this chapter should be answered and discussed, including plots from your simulations that illustrates the conclusions. Using the results, for each case you should try to draw a sketch of the **E**-field behaviour as experienced by a distant observer (in the wave-zone) as a function of time. Also, when appropriate, sketch how the observed spectra (the Fourier transform of **E**(t)) looks.

4.1 Case 1a: Instantaneous deceleration to rest

Investigate the appearance of the electric field for a slowly moving (i.e. non-relativistic) charge that decelerates to rest instantaneously. Experiment with different initial velocities and the number of field lines. Now, how about a charge with a relativistic initial velocity that suddenly stops? Is this case evidently different than the non-relativistic case? If so, what is the reason for this difference?

4.2 Case 1b: Instantaneous change of velocity

Now investigate what happens if the charge changes velocity instantaneously. Experiment with different initial velocities, final velocities and angles. How about if the speed is constant but the angle changes? What physical radiation process does this represent?

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[1]Written by Magnus Gålfalk in 2007

### 4.3 Case 2: Oscillating charge

Try simulating a charge that oscillates at non-relativistic and relativistic maximum velocities. Experiment with the settings - using a small oscillation radius is a nice way to get an overview of the electric field. Does this represent a real physical process, if so, which one?

4.4 Case 3: Circular motion

Circular motion of electrons is common in astrophysics when magnetic field lines are present. Simulate a charge in circular motion at low, medium and high velocities. Experiment with the number of field lines shown and the radius of the circle. What physical process(es) does this radiation correspond to?

4.5 Case 4: Fun with spline curves (optional)

Now, this is an optional case which does not have to be included in the report, it is however fun and very illustrative. Play around with the settings and draw a few curves for the charge to follow at different velocities. Can you draw an elephant?
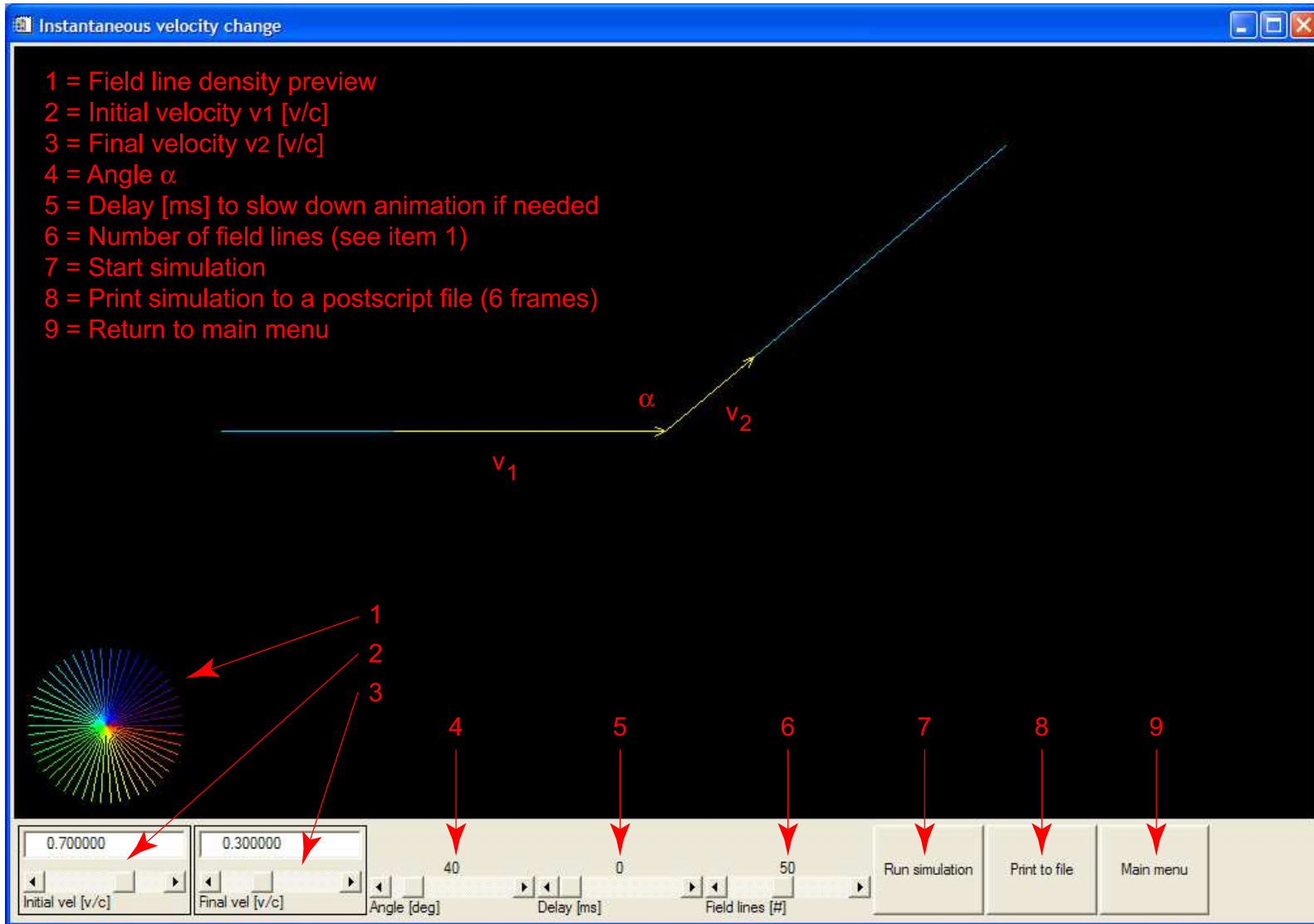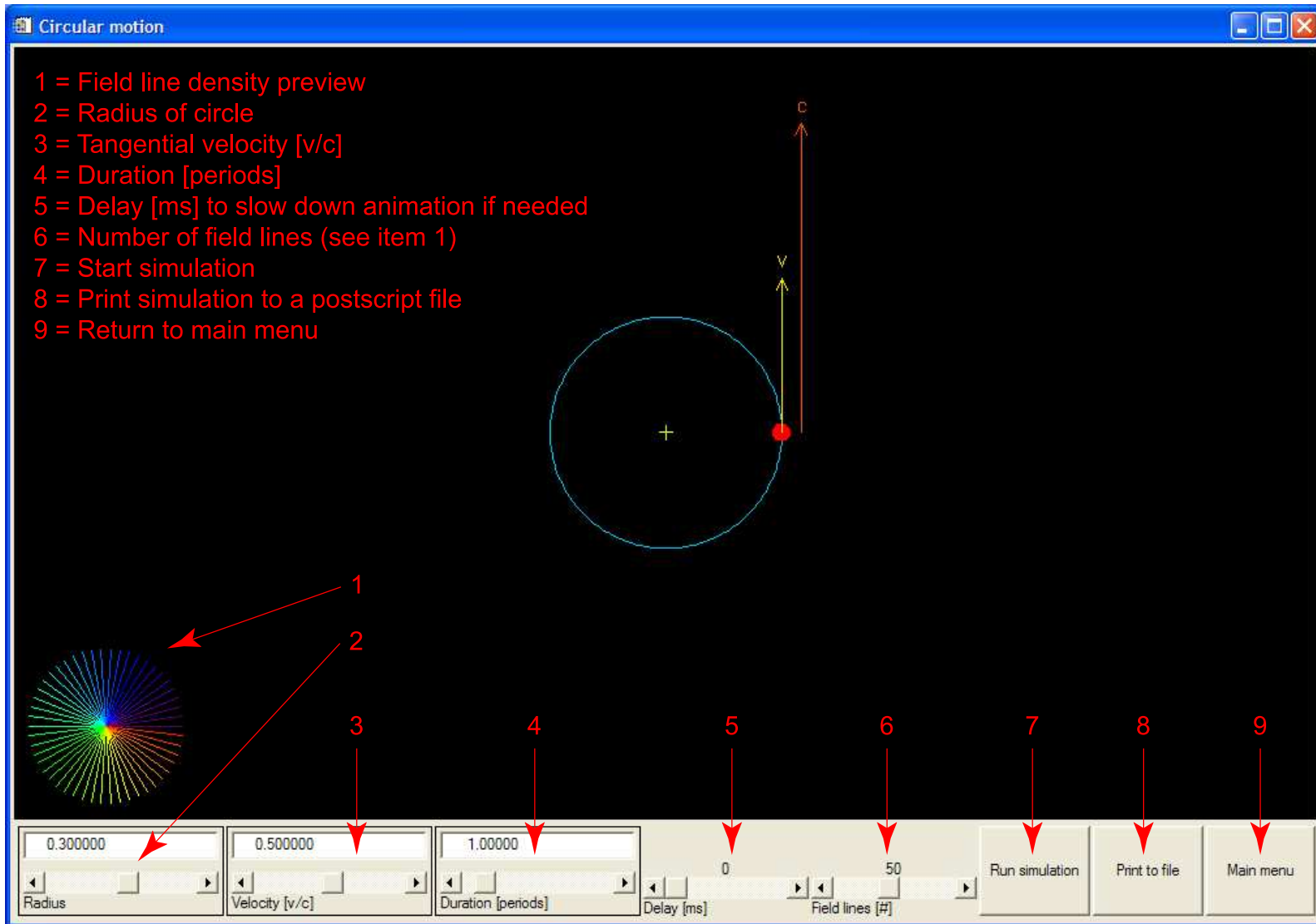
Figure 3: User interface for **instantaneous velocity change**. After setting the physical parameters (2–4) and display settings (5–6) the simulation can be run in real time (7) or printed to a postscript file (8). To get back to this interface after a simulation, click left mouse button anywhere in the window.
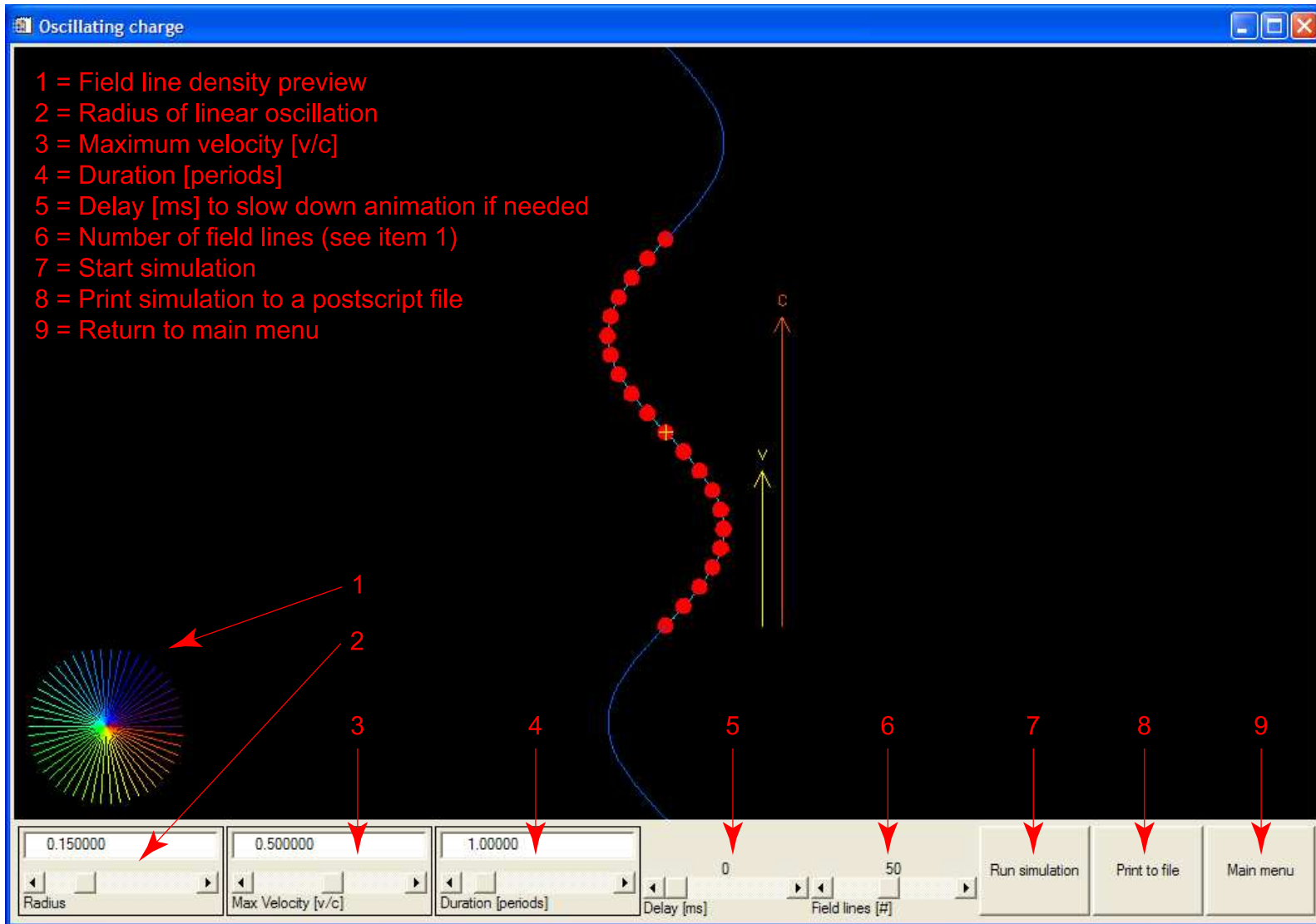
Figure 4: User interface for **oscillating charge**. After setting the physical parameters (2–3) and display settings (4–6) the simulation can be run in real time (7) or printed to a postscript file (8). To get back to this interface after a simulation, click left mouse button anywhere in the window.

Figure 5: User interface for **circular motion**. After setting the physical parameters (2–3) and display settings (4–6) the simulation can be run in real time (7) or printed to a postscript file (8). To get back to this interface after a simulation, click left mouse button anywhere in the window.
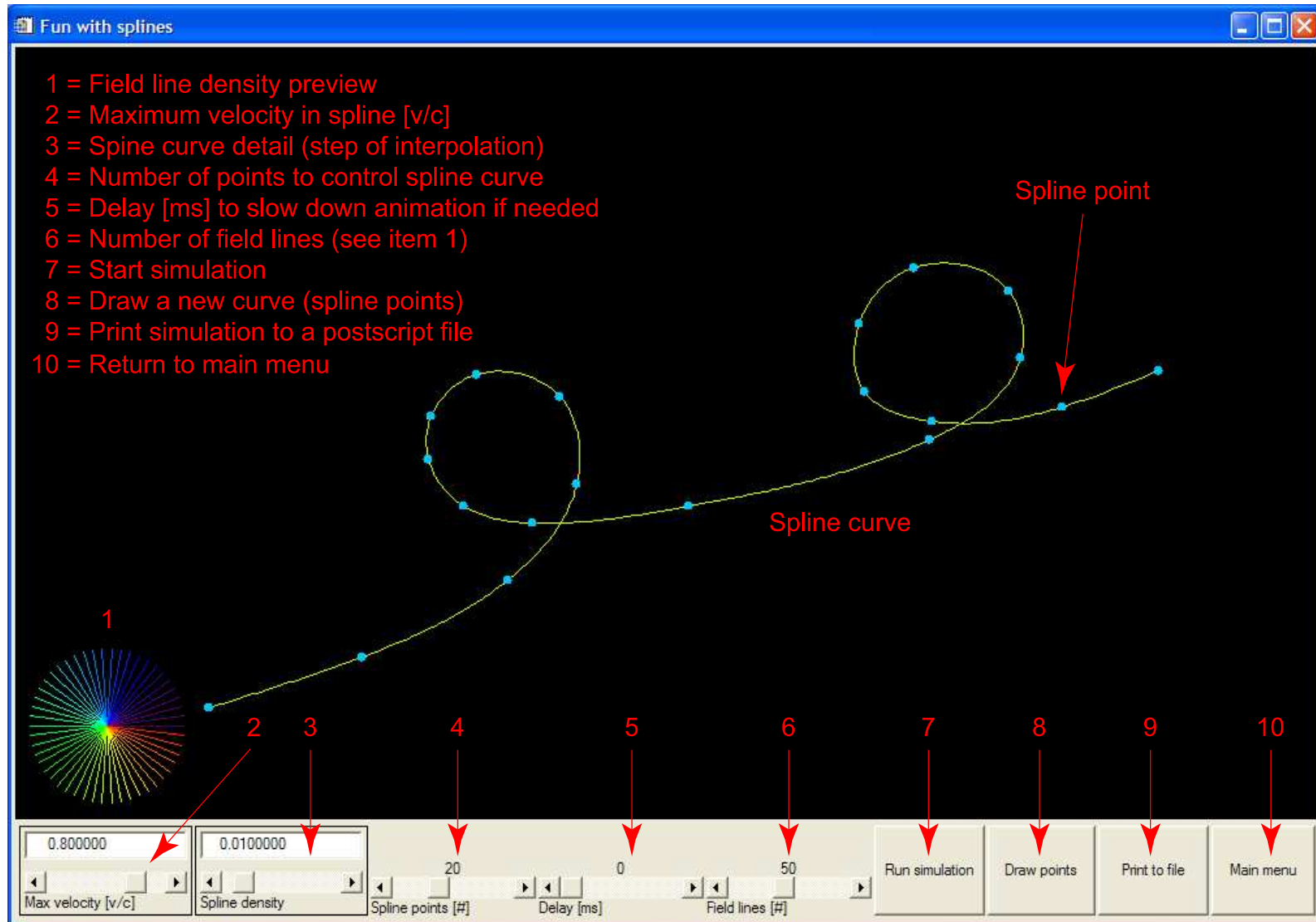
Figure 6: User interface for **fun with splines**. After setting the physical parameters (2) and a motion path has been drawn (4 and 8) the simulation can be run in real time (7) or printed to a postscript file (9). The display settings can also be changed (3, 5 and 6). To get back to this interface after a simulation, click left mouse button anywhere in the window.