Stellar and emission line spectra

Laboratory exercise - Astrophysical Spectra

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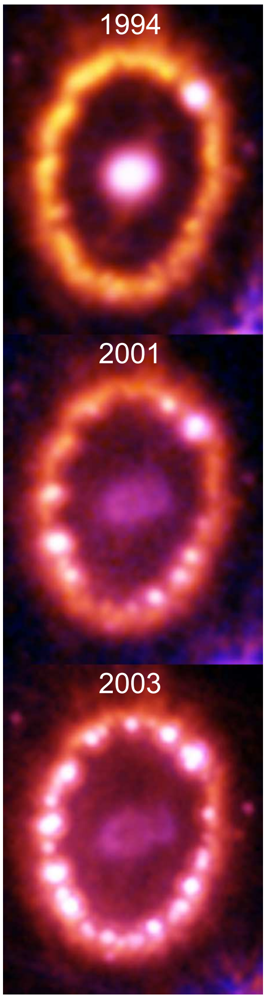
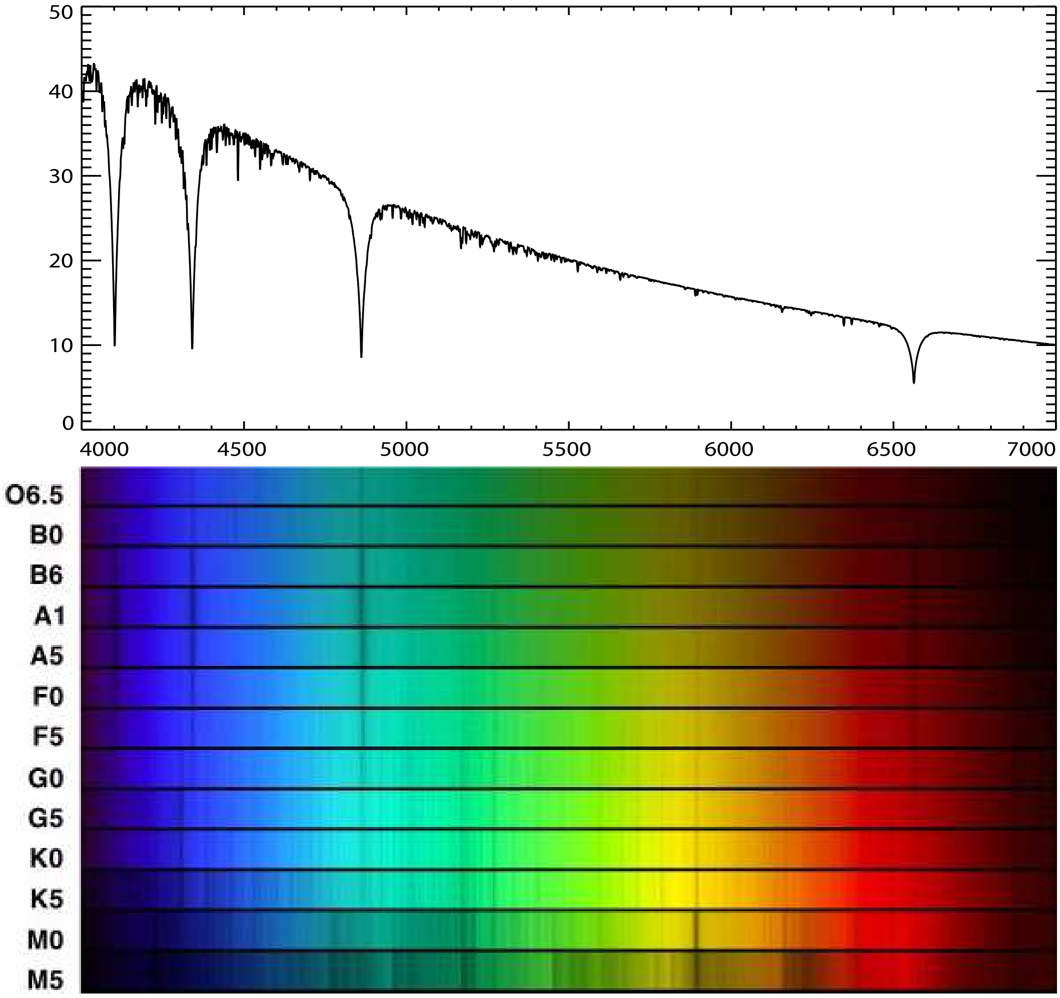
Stockholm Observatory 2007





1 Introduction

Almost everything we know about the universe outside our solar system comes from information provided by light, or more precisely, electromagnetic radiation. The science of astronomy is thus intimately connected to the science of analysing and interpreting light. The purpose of this exercise is to provide some insight into how information can be extracted from observations of spectra. Several spectra are studied; first from stars of various types, then a spectrum from the nebula surrounding the supernova remnant SN 1987A (in the Large Magellanic Cloud). Sect. 2 introduces some background theory and general concepts that are used in the exercises. Sect. 3 gives some preparatory exercises. Sect. 4 shortly describes the functionality of the program *Spectrix*, a tool to assist in the analysis of the spectra. In the final section, the exercises to be solved during the lab are detailed.

1.1 The report

To pass the lab you must write a report, preferably to be handed in within two weeks of the lab. Regarding the level of detail in the report, a good rule of thumb is that the report should be comprehensible to yourself even in five years from now. Answer every exercise fully, including the preparatory exercises, and be careful to estimate errors in your figures when asked to. If you wish, you may write in English, although not a requirement, it is always a good practice.

2 Background theory

2.1 Radiation processes

Radiation can be both emitted and absorbed in a number of processes. Here we concentrate on mainly two processes: free-free and bound-bound. "Free" and "bound" refer to the state of a particle, in our case an electron, interacting with a photon. The electron is considered to be bound if it is bound to an atomic nucleus, and free if it is not. Thus a process is called "free-free" when the electron is free both before and after the energy exchange with a photon, and "bound-bound" correspondingly. As you may have guessed, there are also "bound-free" processes etc.

If bound, the electron can only exist in particular states with particular energies. According to quantum mechanics, the possible states of the electron has energies $\{E_0, E_1, E_2, \ldots\}$, where $E_n < E_{n+1}$. This is called to be a discrete set of energies, as opposed to a continuum set of energies; in a discrete set, there are always energies E such that $E_n < E < E_{n+1}$, for all n. It is sometimes useful to draw an energy level diagram. In Fig. 1, a schematic energy level diagram is shown.

By switching energy level (called a level transition), an electron can either absorb the energy of a photon, or produce a photon with the excess energy from the transition. Since the energy of a photon is inversely proportional to the wavelength of the corresponding light wave, $\Delta E = hc/\lambda$ (where $h = 6.6261 \times 10^{-34}$ J s is Planck's constant, $c = 2.9979 \times 10^{8}$ m s^{-1} the speed of light, and λ the wavelength), a level transition gives rise to an absorption/emission line at the wavelength corresponding to the energy difference between the two levels (Fig. 1). Every transition thus has a particular wavelength $\lambda = hc/\Delta E$ associated with it, and by studying light at that wavelength,

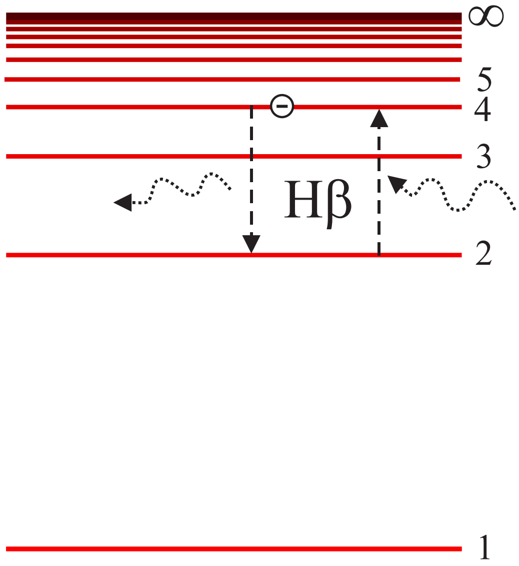


Figure 1: A level diagram of the hydrogen atom. The energy difference between two adjacent levels clearly decrease with level. Since the energy of level n is given by E_n = -13.6/n^2 ev (-13.6, -3.4, -1.51, -0.85, -0.54, ...) this means that a hydrogen atom is close to being ionized already at about level 5–6. The *Balmer* series involves transitions up from or down to level 2, and is denoted with **H** followed by α, β, γ, ... for increasing upper level. Thus, e.g. Hβ means a level 2–4 (absorption) or 4–2 (emission) transition.

we may be able to deduce something about the physical conditions of the responsible matter (see Sect. 2.3).

If a free electron is involved in the transition (free-bound, bound-free or free-free) there are no discrete energies the free electron has to adhere to. Consequently, the photons from such processes may be of any wavelength, a so-called continuous distribution.

2.2 Properties of matter in thermal equilibrium

Atoms and ions with bound electrons can change their state by collisions with other particles. If these collisional transitions are more frequent than the radiative transitions, then the levels populated by electrons will be determined by the statistical properties of the collisions. A special kind of statistical state of matter is called thermal equilibrium; that is the state a closed system of particles eventually will tend to. When the system is not closed, but closed enough to reach equilibrium before disturbed, the state is called Local Thermal Equilibrium (LTE). In LTE, electrons in bound states are distributed among levels according to the Boltzmann equation:

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} \exp\left(-\frac{E_j - E_i}{k_B T}\right), \tag{1}$$

where n_j/n_i is the relative population of electrons in level j compared to level i, g_j and g_i are statistical weights of those levels, T is the temperature (in Kelvin) and $k_B = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant. For any level i of hydrogen, the statistical weight and level energy can be written $g_i = 2i^2$ and $E_i = -13.6/i^2$, respectively. *The level population thus only depends on the temperature, a very important and often used property of LTE.*

Gas in LTE emits light due to free-free processes, with a spectrum according to the Planck distribution:

$$B_\lambda \propto \lambda^{-5} \left[\exp\left(\frac{hc}{\lambda k_\mathbf{B} T}\right) - 1\right]^{-1}, \tag{2}$$

where B_λ is the intensity of light as a function of wavelength. This distribution (also called black-body radiation) also depends solely on temperature.

If the temperature is high enough, the collisions between the atoms may be energetic enough to strip the atoms of electrons; that is, the atoms get ionised. The ions then later recombine with a free electron to become a neutral atom again. In thermal equilibrium, the number of ionisations and recombinations are balanced and well described by the so-called Saha equation. This equation describes what the degree of ionisation is for a particular temperature (see preparatory exercise 2 below); the higher the temperature, and the lower the density, the more strongly particles become ionised. As an example, at typical densities of a stellar atmosphere, about half of the hydrogen becomes ionised at 10 000 K.

2.3 Line formation

This section concerns how spectroscopic lines are formed, and why they are seen in absorption sometimes, and in emission at other times. The simple picture of a cold gas being illuminated by a background source (Fig. 2) is seldom valid in astrophysical applications, as the gas itself often either is warm and gives rise to line emission (as in a stellar atmosphere) or is strongly ionised and produces *recombination line emission*. The nature of recombination line emission is reviewed in Sect. 2.3.2.

2.3.1 Stellar atmospheres

A star, being a fuzzy ball of hot gas, does not have the same kind of well-defined surface as, for instance, a rocky planet. Instead, the density of a star drops more or less smoothly from the centre outwards, approaching the interstellar density far out from the star. The radius of a star is merely a matter of definition. Fortunately, there is a sharp transition region in stars that can serve as the defining radius, and that is the photosphere. The photosphere is the region where the star, as seen from an outside observer, suddenly becomes optically thick, i.e. non-transparent. The radius of the photosphere (and hence the star) is only a weak function of wavelength, so most photons we see come from approximately the same region in the star. The photosphere is the region where the continuum from a star is emitted, the radiation due to free-free processes in the hot gas that is well approximated by blackbody radiation.

Even though the star suddenly becomes essentially transparent above the photosphere, there is still a substantial mass of gas further out. The gas may be too sparse for free-free processes to be efficient, but bound-bound processes are certainly still important, though limited to certain wavelengths in contrast to free-free processes (bound-free processes are also important, but we do not consider them here). This means, that while photons from the photosphere generally travel unhindered to the observer, photons that happen to reside at wavelengths where important bound-bound transitions prevail, only reach a certain distance l on average before they are absorbed by the stellar atmosphere (Fig. 3). This variable, l, is usually called the mean free path of the photon, and depends on how strong the transition is in an absorbing species (specific atom, ion or molecule) and its density. The higher the probability that a photon will be absorbed by a single species, and the higher the density is of that species, the shorter the mean free path of the photon becomes.

As the density of absorbing particles drops further out from the star, the mean free path becomes longer, and eventually long enough for the photon to travel more or less freely to the observer (Fig. 3). The radius from the stellar centre at which this happens depends on the

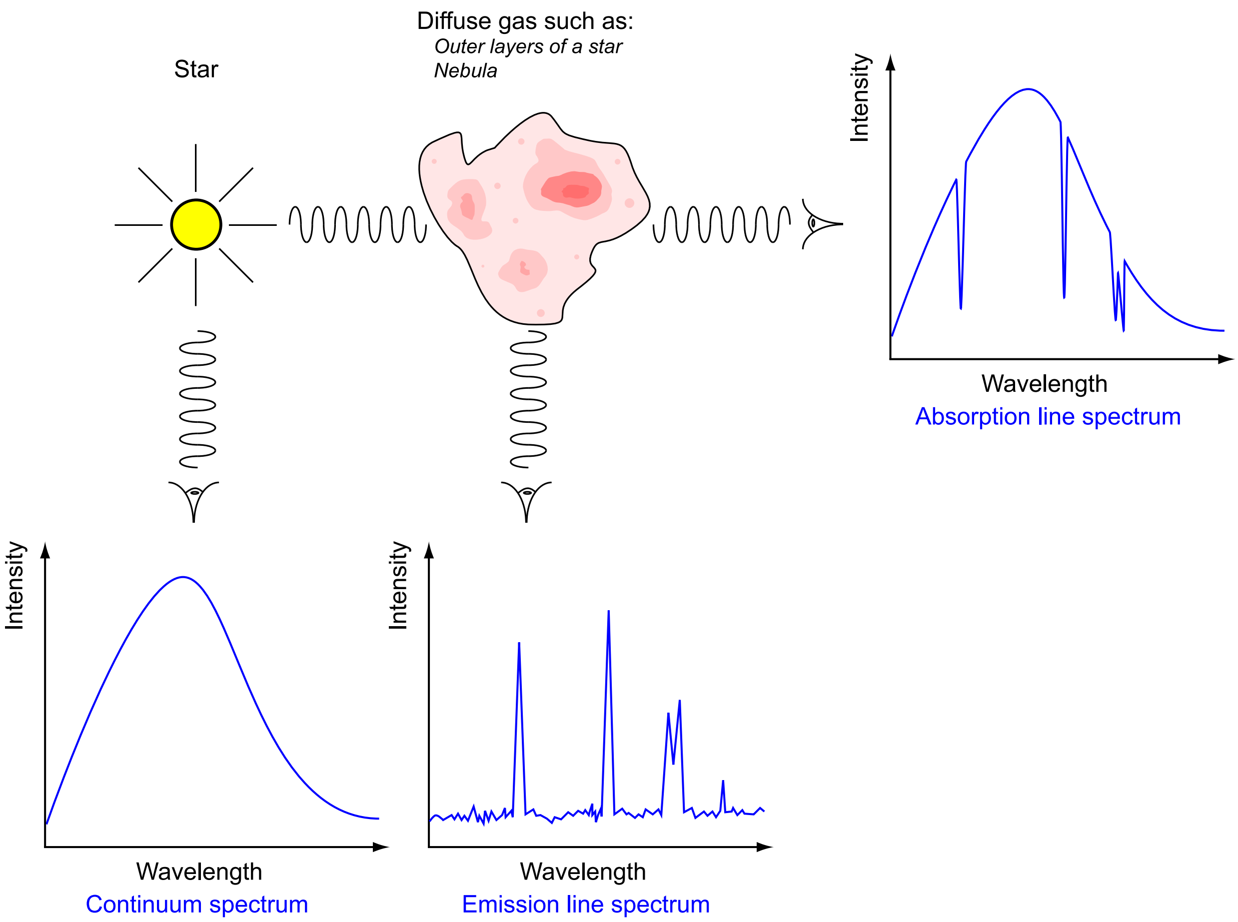


Figure 2: The light from a continuum source (the photosphere of a star) gives rise to absorption lines when observed through diffuse gas (such as the outer layers of a star or a nebula). However, if an illuminated nebula is instead observed from the side (starlight being absorbed and re-emitted by the gas in another direction) an observer would see emission lines.

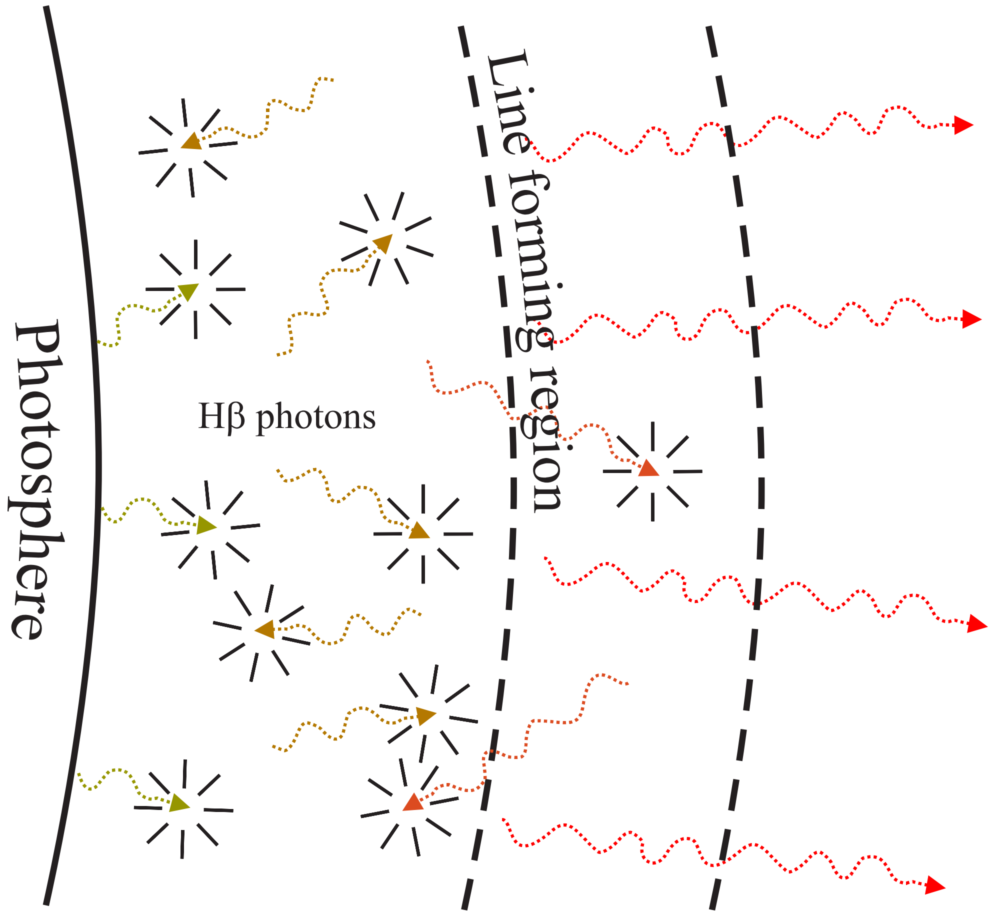


Figure 3: When a photon at a wavelength corresponding to a strong bound-bound transition (like Hβ in this Figure) leaves the photosphere, its mean free path is short and it is soon absorbed. However, as the density of the absorbing particles decrease outwards, the mean free path grows to ultimately become essentially infinite. The flux intensity in the line at that point depends on the local temperature of that region. As shown in Fig. 4, depending on that temperature compared to the continuum temperature, we can get an emission (hotter) or absorption (colder) line. This is the reason that absorption lines are much more common than emission lines in stellar spectra, given that the temperature usually decrease outwards in stars.

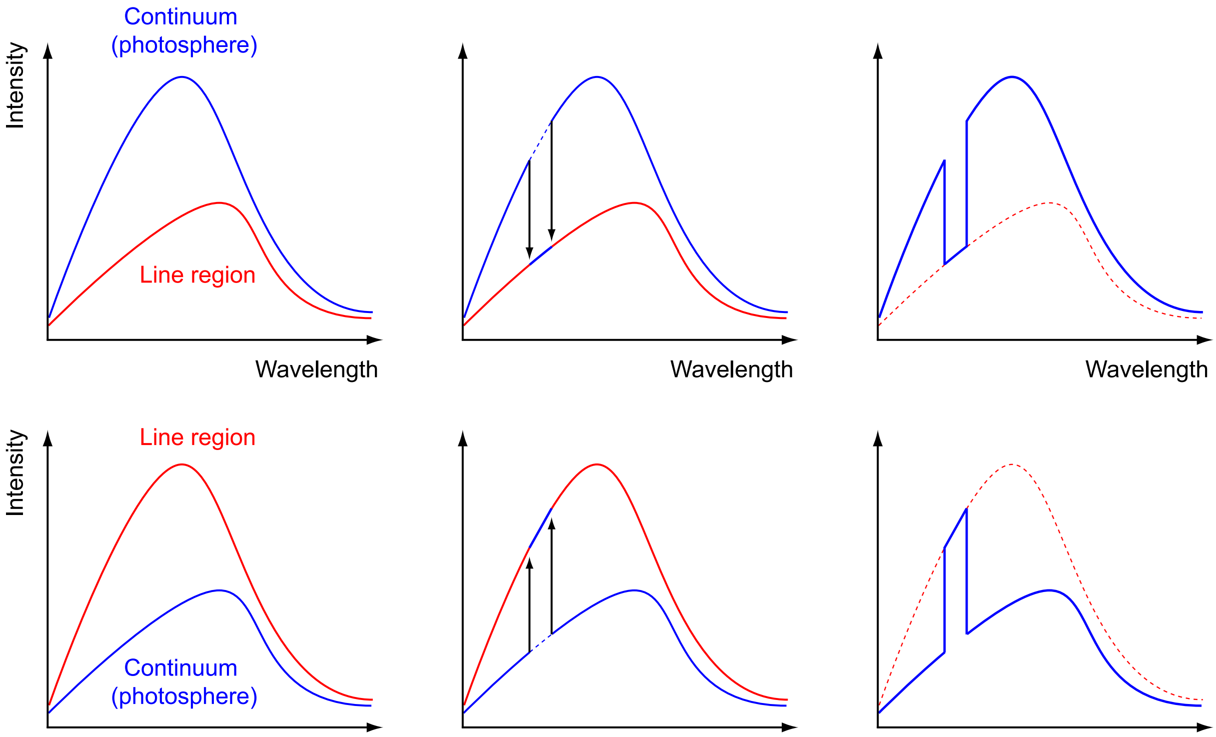


Figure 4: The principle behind absorption and emission lines in stars. The upper row results from a temperature-radius profile like the one in the left panel of Fig. 5, where the temperature of the continuum (blue curve) is higher than in the line forming region (red cruve). The continuum arise in the photosphere of a star and is essentially a blackbody curve with a temperature T_{eff}. Further out from the star (where the line is formed) light is absorbed, then emitted again but with a lower intensity because of the lower temperature (a blackbody curve with a lower temperature has lower intensity at all wavelengths). This partial replacement of a blackbody curve with a lower intensity one creates an absorption line as seen in the spectrum by an observer (see rightmost panel). The situation is similar for an emission line (bottom row), however, in this case $T_{eff} < T_{line}$ (as in the right panel of Fig. 5) which means that the absorbed light is replaced with the higher intensity of the hotter blackbody curve. Note that, for clarity, the scale is very exaggerated in this Figure, real spectal lines are much thinner.

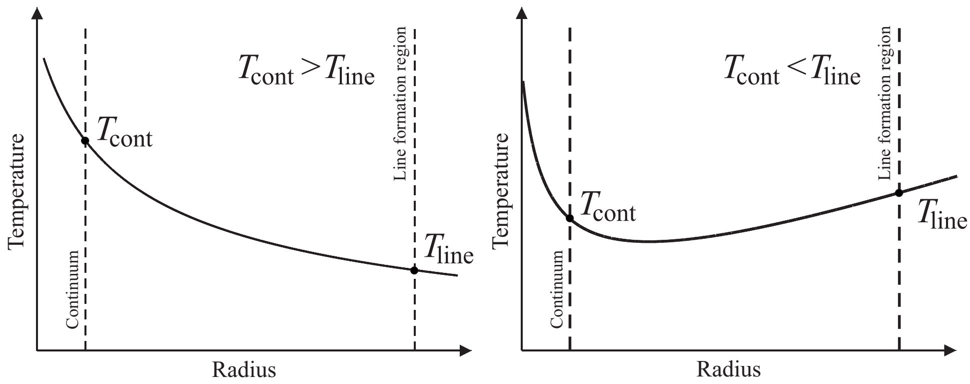


Figure 5: The figures show examples of temperature profiles in stellar atmospheres, where the radius increases to the right. If the continuum forms in a region of the atmosphere where the temperature is higher than the region where the line is formed, then we get absorption (left panel). In contrast, if the photons we see in the line come from a region where the temperature is higher, we get an emission line (right panel). Se also Fig. 4.

particular transition, so it varies strongly. This means that the photons an observer sees come from different layers in the star. If the local temperature of the layer is lower than the layer where the continuum is emitted, then the LTE emission in the line is lower, and an absorption line is seen (upper row, Fig. 4). If, on the other hand, the layer temperature is higher, then an emission line can arise (lower row, Fig. 4). In a real stellar atmosphere, the radial temperature structure is fairly complicated and gives rise to line emission both below (absorption) and above (emission) the background continuum, depending also on the detailed ionisation structure.

2.3.2 Gaseous nebulae

The reason why the gas in stars is kept in (or at least close to) thermal equilibrium is that densities are high. Interstellar gas clouds, however, are typically so sparse ($n_e \sim 10^6 - 10^{11}$ m^{-3}) that collisions between particles are very rare. This gives time for radiative transitions to re-assemble the energy levels of the atoms and ions so that they no longer are distributed according to the Boltzmann equation. Instead, particles are expected to reside most of their time in the state of lowest energy. For these objects thermal equilibrium is far from a good description, and the emitted spectrum is very unlike a blackbody spectrum attenuated by absorption features. Instead, the spectrum is dominated by *emission* lines sitting on top of a very weak continuum. If there is a hot star radiating energetically enough to ionise atoms in the neighbourhood, then several of the species may be strongly ionised, and the occasional capture of an electron by one of these ions gives rise to recombination radiation, as the electron cascades down to the ground state, emitting radiation on its way. Another source of emission is by scattered (or *re-emitted* light), where photons from a nearby source excite particles of the nebula that subsequently re-radiate the photons, but in a possibly different direction and sometimes at a different wavelength. Finally, if there is a background source, one may see the gas in absorption against the source, much like in Fig. 2; this is commonly seen against stars as interstellar absorption lines, and may be distinguished from the atmospheric absorption lines of the star by their comparatively small line width, and usually different radial velocity relative to the star (i.e. the interstellar lines are

Doppler shifted differently than the atmospheric lines). One good thing about gaseous nebulae is that it is relatively simple to use them to derive abundances of elements like H, He, C, N, O, Ne and S.

2.4 Line broadening

Since bound-bound processes give rise to a photon with the specific energy of the difference between the transitioning energy levels, one might expect that the photon would always have exactly this energy, and that the corresponding spectral lines thus would be infinitely sharp. This is clearly not the case, and for several reasons. First, due to fundamental quantum mechanical limitations, the energy can never be exact, but must have a measure of uncertainty associated with it. Secondly, and usually more important, since the interacting particles are almost never at rest relative to the observer, the energy of the photon and thus its wavelength, will be Doppler shifted. If we add up light coming from particles moving in different directions, this will effectively broaden the spectral line. One example when this happens is the intrinsic thermal motions of the particles, another is the rotation of a star, where different parts of the star move with different velocities relative to the observer, and thus gives rise to a rotationally broadened line.

3 Preparatory exercises

In order to do the actual computer exercise, **it is important that the preparatory exercises below are completed** (or at least seriously attempted). This is mainly to ensure that sufficient preparations for the lab has been made and to allow for the best use of the lab time. The first three exercises are mostly needed for a theoretical understanding of stellar absorption lines, while the result of the fourth exercise is needed in one of the lab exercises.

The strength of an absorption line depends mainly on the temperature and its gradient in the stellar atmosphere. The following two problems are intended to illustrate the role of these physical parameters for the formation of an absorption line. The first problem focuses on the temperature gradient by making the (unrealistic) assumption that the number of atoms that contribute to the absorption be independent of the temperature. In the second, and more realistic problem, the temperature gradient is kept fixed. Instead, it is emphasised that the fraction of all atoms that contributes to a spectral line is determined by the temperature through Saha's equation. Henceforth we will use n_I to denote the total number density of all hydrogen atoms with bound electrons (i.e. $n_I = n_1+n_2+...+n_\infty$). In the case of the H$\beta$ line, not all hydrogen atoms have an electron in the appropriate level (i.e., the principal quantum number n = 2, denoted n_2). The electron can be in any other level or even separated from the proton (ionisation). We will use n_{II} to denote the number density of ionised hydrogen atoms and $n_{tot} = n_I + n_{II}$ for the total number density of all hydrogen, neutral and ionised.

1. The aim of this problem is to show how an absorption line forms, and that different temperature gradients lead to different strengths of the absorption line. In Fig. 6a we plot the density (ρ) as a function of radius for the gas in the region where the Balmer lines are formed. In order to simplify the mathematics, ρ is assumed to be constant.

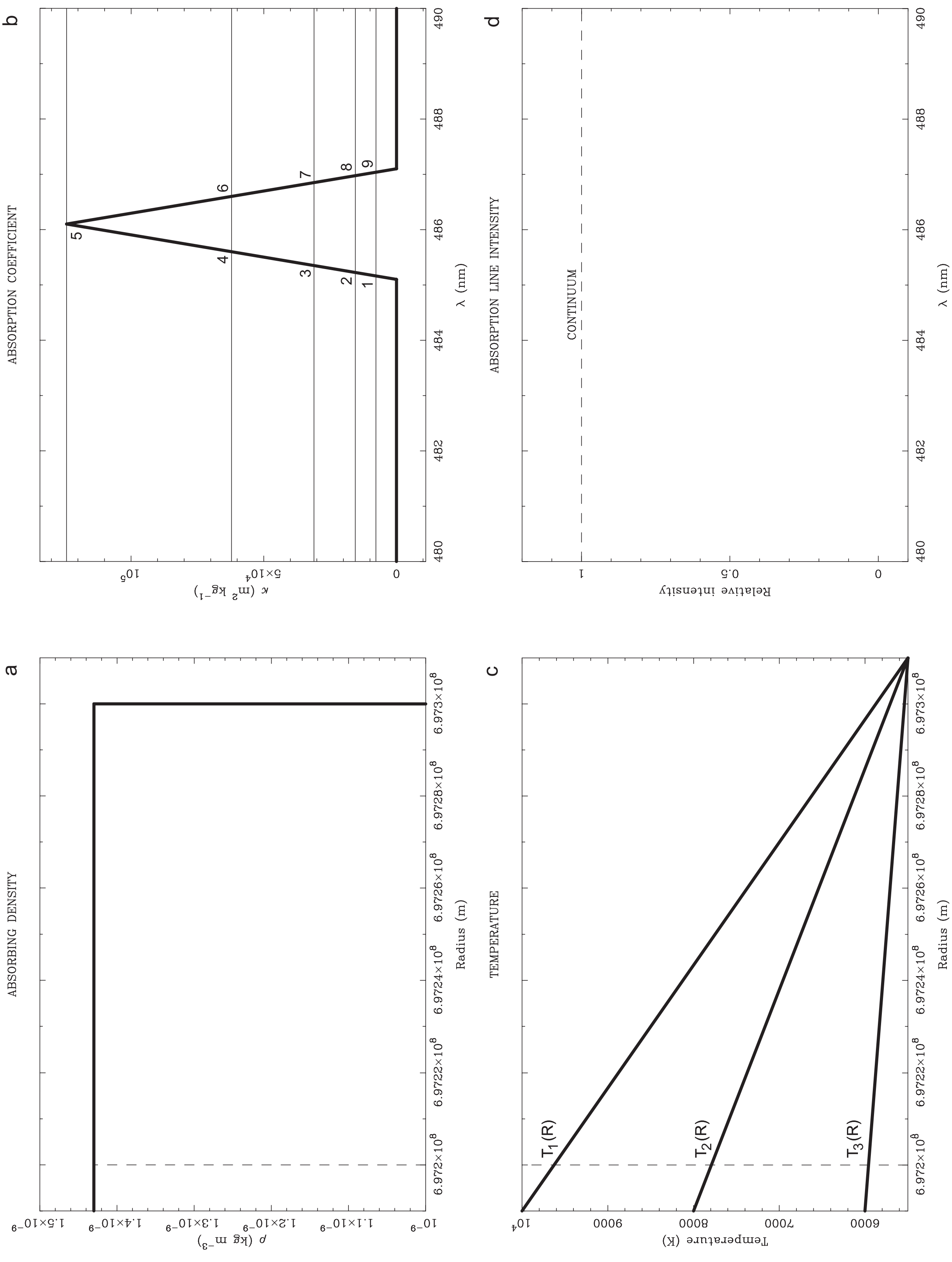


Figure 6: Plots for preparatory exercise 1

Furthermore, the star is assumed to be of pure hydrogen. Only the outer part of the star, i.e., the absorption line region, is represented in the diagram. The dashed line marks the beginning of the photosphere, i.e., the radius from where the continuum radiation is emitted. In Fig. 6b we show the mass extinction coefficient (sometimes denoted as the absorption coefficient) for Hβ as a function of wavelength. In this plot we have also drawn five horizontal lines, each representing the value for the mass extinction coefficient at the two wavelengths where the horizontal line intersects the mass extinction coefficient profile. In Fig. 6c we plot three different temperature gradients in the stellar atmosphere to simulate three different stars.
Before we start, we need to introduce a new quantity, the *mean free path*, l, which gives the average distance a photon can travel before being absorbed. It is defined as

$$l = \frac{1}{\kappa\rho} \tag{3}$$

where κ is the mass extinction coefficient. We proceed as follows:

- Read the values of the mass extinction coeffcient κ for the corresponding wavelengths. Compute the mean free path of the photons according to Eq. 3.
- Subtract the obtained mean free path from the stellar radius in order to obtain the depth in the star from where the observed photons emerge. The five different values of κ correspond to five different mean free paths, and thus five different depths.
- For each of these depths, estimate the temperature for the three different temperature gradients.
- Compute the blackbody radiation intensity from these various depths using the Planck function
$$B_\lambda = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{(hc/\lambda kT)} - 1}\right) \tag{4}$$
It is sufficient to calculate the relative intensities, i.e., to divide the obtained intensities by those of the continuum for the respective stellar model. Plot the relative intensities in Fig. 6d.

Your results should show three different absorption lines, of which the strongest is associated with the steepest temperature gradient.

2. To illustrate the temperature dependence of the level population/ionisation in a stellar atmosphere, we consider the number density in the ground state of hydrogen, n_1. In the case of thermodynamic equilibrium (T.E.) this is related to the number density of protons, n_p, according to the *Saha equation* (note that the Saha equation usually is written as the inverse of this)
$$\frac{n_1}{n_p} \approx \frac{n_I}{n_{II}} \approx 4.14 \times 10^{-22} n_e T^{-3/2} e^{\chi_I/kT} \tag{5}$$
where we have the assumption that most bound electrons are in the ground state, n_e is the electron density in m^{-3}, T the temperature in K, $\chi_I = 13.6$ eV the ionisation threshold of

level 1 and Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}. Assume $n_e = 10^{21}$ m^{-3} and calculate for what temperature $n_1 = n_p$. (1 eV = 1.6022×10^{-19} J). Note that the equation has no analytical solution. It has to be solved by iteration, using for example the Newton-Raphson method. It can also be solved graphically. Assuming the *total* stellar density to be constant (see Fig. 7a) we plot in Fig. 7b three different stellar models with the same temperature gradient, but with different temperatures in the region where the continuum arises.

To account for variations of n_2/n_{tot} with temperature we should use, instead of Eq. 3,

$$l = \frac{1}{\kappa\rho_2} = \frac{1}{\kappa\rho}\left(\frac{n_{tot}}{n_2}\right) \tag{6}$$

where n_2/n_{tot} is given by an expression similar to Eq. 5 and is plotted in Fig. 7c.

The mean free path, for Hβ photons say, can now be computed for the different temperature structures by assuming that the density of the absorbing material is constant, but different for the individual temperature structures. The constant value is taken as the mean value in the absorption line region. The mean values are given below, together with the temperatures of the continuum photosphere:

$T_{C1} = 8800$ K, $< n_2/n_{tot} >= 9.82 \times 10^{-7}$
$T_{C2} = 12800$ K, $< n_2/n_{tot} >= 1.50 \times 10^{-5}$
$T_{C3} = 17800$ K, $< n_2/n_{tot} >= 4.56 \times 10^{-6}$

Calculate from Eq. 6 the mean free path for photons in the line centre using the mass extinction coefficient κ given in Fig. 6b. Use this value to determine (just as in the previous problem) the depth in the atmosphere from where Hβ radiation can escape and, thus, be observed by us. Estimate, from the figure, the temperature at these depths for the three stellar models respectively.

A crude estimate of the relative strengths of the absorption line in the three different stars can be made by computing the difference between the blackbody intensity at the continuum photosphere, B_C, and the blackbody intensity at the depth from where the Hβ photons emerge, $B_{H\beta}$. Normalise this difference by dividing with B_C to obtain an estimate of the absorption line strength.

$$f_\lambda = \frac{B_C - B_{H\beta}}{B_C} \tag{7}$$

Do this, and plot the ratio as a function of the temperature of the continuum photosphere. How do you think this ratio would change for lower (T ≤ 9000 K) and higher (T ≥ 18000 K) temperatures, respectively?

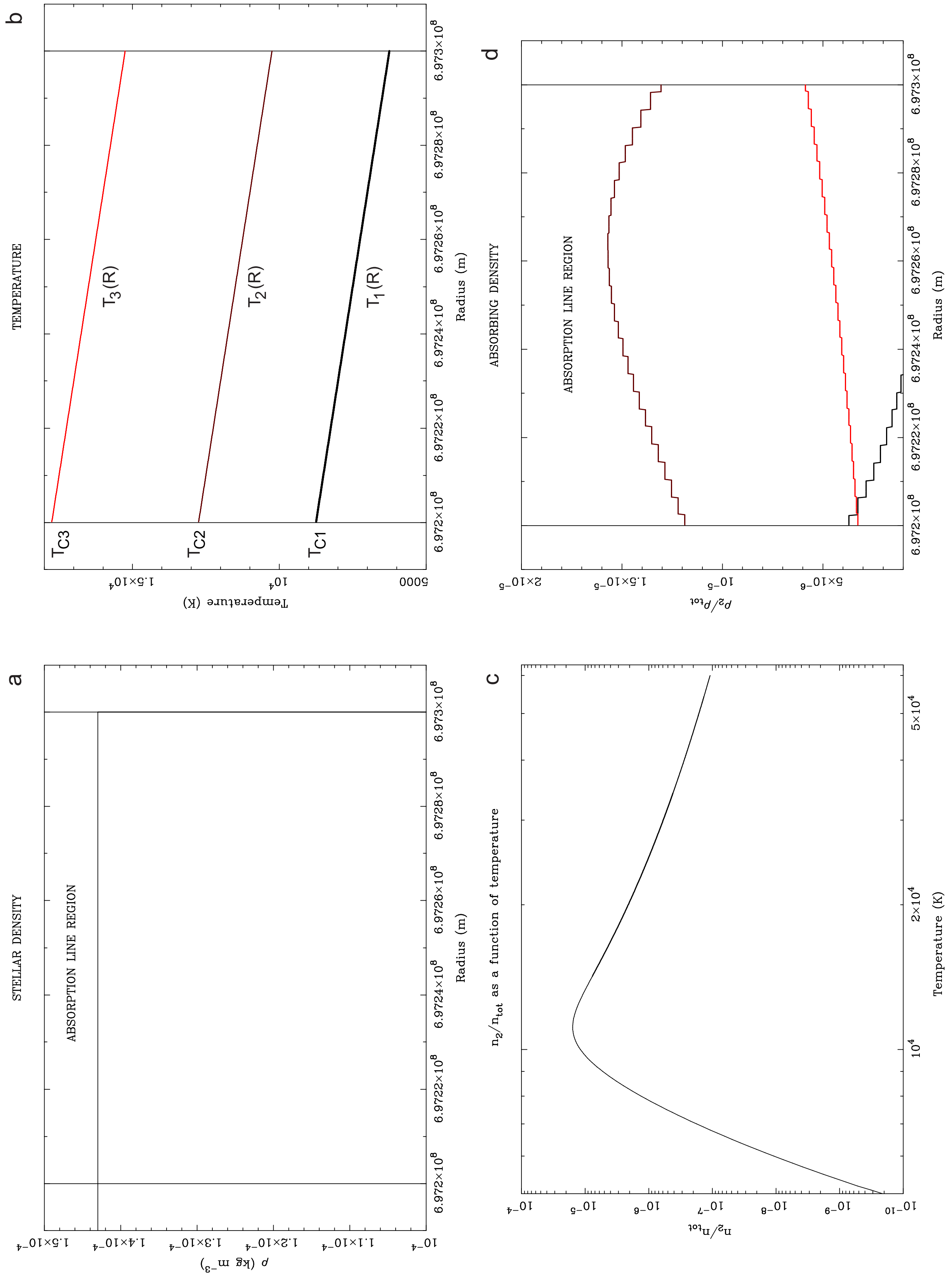


Figure 7: Plots for preparatory exercise 2

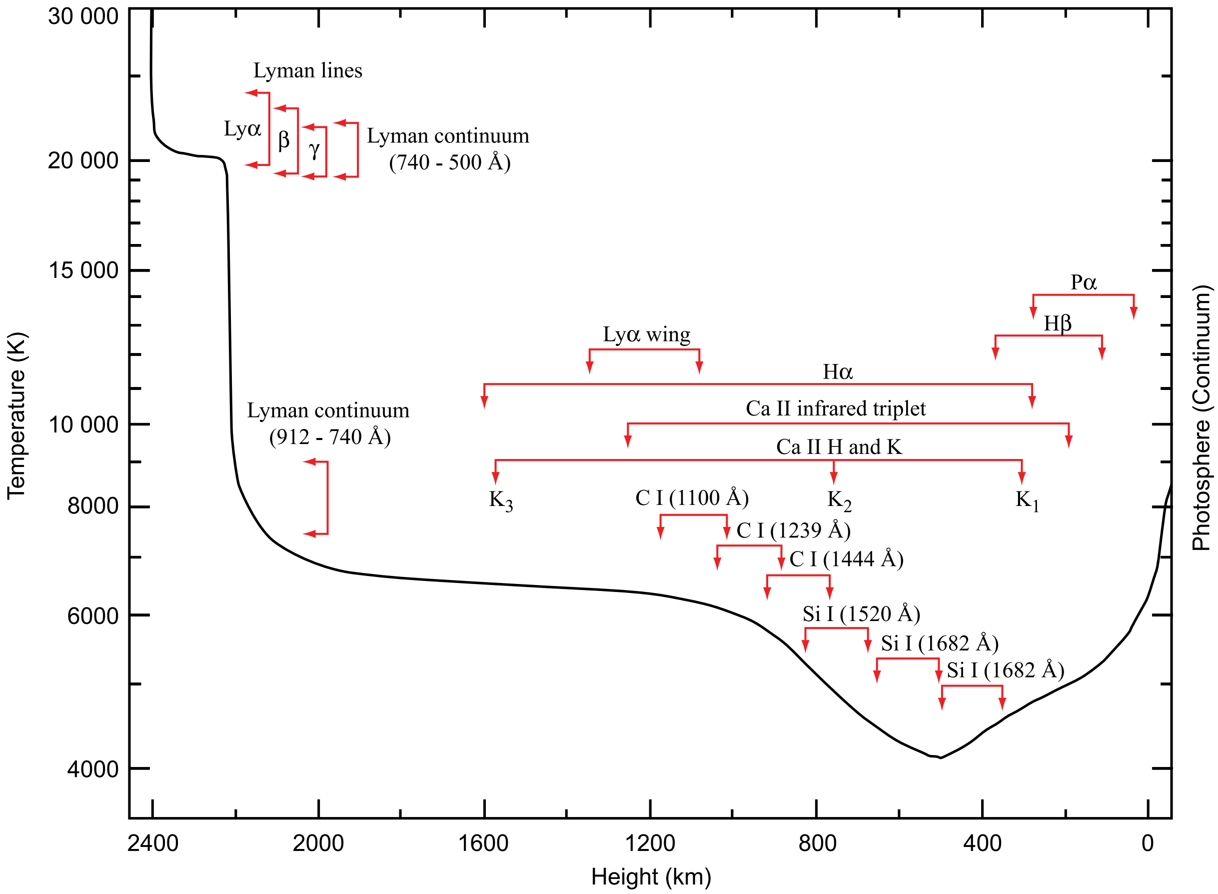


Figure 8: Temperature structure of the Sun with important line forming regions marked. The height, *h*, is put to zero roughly at the photosphere. Thus, at zero height we have a contiuum distribution, similar to a black body with a temperature of $T_{eff} \approx 5800\,K$.

Note that normally the strength of an absorption line is given as the area of the absorption line below the continuum. See Fig. 11 for the definition of the so called equivalent width.

3. The Planck function (Eq. 4) has the property that when λ is kept fixed, it increases monotonically with T, so that

$$\frac{dB_\lambda}{dT} > 0. \qquad (8)$$

This has important consequences for stars; if some parts of the spectrum are emitted by regions cooler than others, this causes dips in the spectrum, seen as absorption lines at these particular wavelengths (see Fig. 4). In general, stars have a temperature that decreases with radius. This is shown in Fig. 8 for the Sun; the temperature drops from $\sim$ 6 500 K at a radius close to the continuum photosphere to $\sim$ 4 000 K, 550 km above this radius.

As can be seen in the figure, solar spectral lines form above the continuum photosphere, meaning that the line forming regions are cooler than the gas emitting the continuum. The situation is similar in other stars, and thus, stellar spectral lines are in general absorption lines. In particular, this is the case for the Balmer lines of hydrogen (transitions from level $n = 2$ to any higher level). Further out in the solar atmosphere ($h > 1000\,\text{km}$) the temperature starts to increase with radius. Lines forming this high up in the atmosphere would therefore be emission lines, however, since the density drops rapidly with radius, the mass of the gas that gives rise to such emission lines is low and these lines are therefore very weak. *Although, during a total solar eclipse, the spectrum of the Sun is in fact mainly an emission line spectrum. Why is this? Hint: What is being observed when most of the Sun is blocked by the Moon?* According to the temperature structure and line forming regions of the Sun given in Fig. 8, which of the indicated lines are expected to show the strongest absorption features?

4. As a healthy exercise, and in preparation for the Spectrix part of the lab, where we are interested in the strength of the Hβ line, we will now estimate n_2/n_{tot} as a function of T in a temperature range valid for most main sequence stars ($\text{T} \lesssim 40\,000\,\text{K}$). This function will look something like the curve plotted in Fig. 7c, however, not exactly the same since it depends on what electron density n_e we choose and the approximations we make.

   In general the ratio can be written as:

$$n_2/n_{tot} = \frac{n_2}{n_I + n_{II}} = \frac{n_2/n_I}{1 + n_{II}/n_I} \tag{9}$$

   For the relatively low temperatures we are considering, compared to e.g. the much hotter interiors of stars, most bound electrons are either in the ground or first excited state, allowing us to make the simplifying assumption that $n_I \approx n_1 + n_2$. Use this approximation to rewrite Eq. 9 to contain only terms of n_2/n_1 and n_{II}/n_I. Finally, use the Boltzmann equation (Eq. 1) and the simplified Saha equation (Eq. 5) to express n_2/n_{tot} as a function of T and n$_e$.

4 Spectrix

Spectrix is a Java tool, designed to aid in the analysis of the spectra in this lab. Fig. 9 shows the layout of the program as it appears on a Windows XP machine, but due to the portable nature of Java, Spectrix will work on other platforms as well, although with a slightly different layout. Spectrix is part of the standard installation package on the student laptops and you can start it by simply typing *spectrix* in a terminal. If you have access to a computer at home, you can download and run Spectrix[1]. Note that you need Java installed as well, but this is usually installed by default on modern operative systems. If you do not have Java installed, you may download the latest Java platform from Sun Microsystems[2].

The main functionalities of Spectrix are:

- Display different parts of a spectrum at different zoom settings.
- Integrate a spectrum over a specified interval.
- Overplot a blackbody function with a temperature set by the user.

The detailed functions of the graphical user interface are described in the caption to Fig. 9.

5 Computer exercises

5.1 Stellar spectra

Depending mainly on the temperature of the photosphere, the spectra of different stars are dominated by different lines. Important lines are shown in Table 1.

The H ϵ line and the Ca II H line nearly coincide in wavelength, at around 3970 Å. The Ca II H & K lines are however usually only seen in cold and medium hot stars (and always both at the same time), while the H ϵ line is only seen in hot stars, so there is usually no problem to distinguish between these two. In addition to these lines, there are also molecular bands due to CN (around λ 414.4-421.5), and TiO (many bands at wavelengths larger than ~ 476.0 nm.), but in this exercise we will focus on the lines given in Table 1.

In the following exercises, we will use spectra to determine properties of stars ranging in temperature from spectral type O5 (hot) to K4 (cold). For one of these stars, having a spectral type of G0 (almost the same as the Sun), we will also investigate the influence on line strengths from varying the surface gravity and the abundance of heavy elements. The surface gravity, log g (cgs units, log $\mathrm{cm\,s^{-2}}$), is 4.5 for most stars in this lab (very close to the Sun value of 4.44). Since a *larger star has a lower surface gravity* this means that giant stars have much smaller log g values than this, while dwarf stars have larger log g. Everything except hydrogen and helium is, in astronomical terms, considered as a "metal". The mass fraction of heavy elements, usually denoted Z, is about 0.02 (2 %) for the Sun. The naming scheme for all stellar spectra in this lab is in the following order: spectral type, log g and finally Z (in cases where $Z \neq 0.02$). Thus, for instance, *A2_4.5* has spectral type A2 and $\log g = 4.5$ while a spectrum called *G0_4.5_0.002* has spectral type G0, $\log g = 4.5$ and $Z = 0.002$ (very low metallicity).

[1] Find Spectrix at `http://www.astro.su.se/utbildning/labb/Spectrix.zip`

[2] `http://java.sun.com`

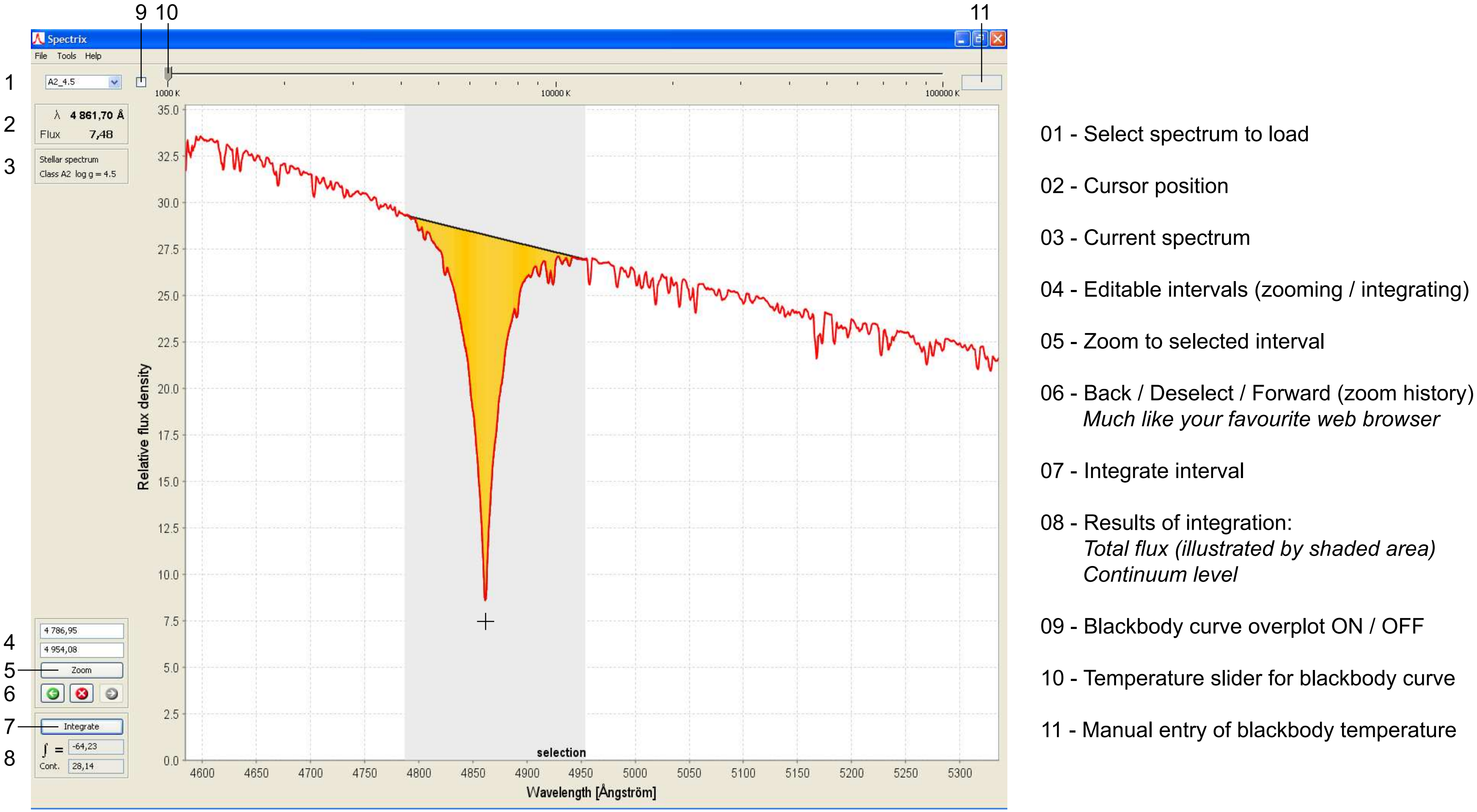


Figure 9: The Spectrix interface. After loading a spectrum (1) it is possible to zoom any part of the spectrum (4–6), indentify a line from its wavelength (2) using the cursor position and to integrate a line (4,7 and 8). A blackbody curve can be overplotted (9–11) in order to make a visual estimate of the effective temperature of a star (T_{eff}).

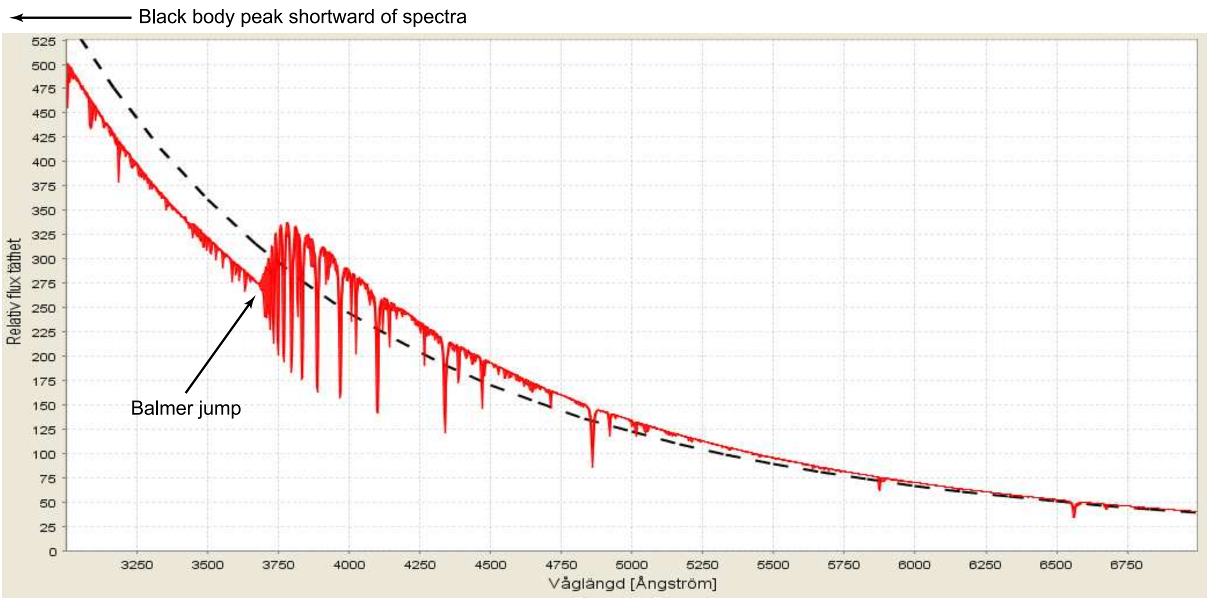


Figure 10: An example of a black body fit to a stellar spectrum. Note the Balmer jump at 3647 Å, corresponding to 3.40 eV, the energy needed to ionise an electron in the $n = 2$ level of hydrogen. Since any photon with a wavelength shorter than the "Balmer edge" has enough energy for the ionisation to occur (all excess energy above 3.40 eV going to the free electron) there is a clear deficiency shortward of the jump.

The effective temperature, T_{eff}, is the "surface temperature" of a star, in the sense that the continuum emitted by its photosphere can be very well described by black body radiation of this temperature. Spectral classes are used to distinguish temperatures, from hottest to coldest the most common types are called O-B-A-F-G-K-M. Historically, the classification scheme was at first in alphabetical order but was then reordered based on temperature. An often used mnemonic for remembering this order is "Oh Be A Fine Girl/Guy Kiss Me". As the shape of the black body curve depends only on its temperature, the shape of a stellar spectrum provides a simple indication of its "surface temperature".

Exercise 1 *For each of the seven spectra with solar metallicity and gravity (O5, B4, A2, F0, G0, G8 and K4), find the black body temperature that best fits the spectrum. An example of a fit is shown in Fig. 10. It might be a good idea to start with the hottest stars in the sample since these are easiest to classify (because of their resemblance to a black body). The reason for this is that colder stars have a lot of absorption bands, especially at short wavelengths, the spectrum is thus deficient of photons in that region compared to a blackbody. Because the energy has to come out somewhere in the spectrum, at longer wavelengths, the spectrum lies above the black body curve (when a decent temperature has been found).*

Start by marking the box left of the temperature slider in Spectrix, and slide the temperature until the shape of the black body curve roughly corresponds to the stellar spectrum. To get a good temperature fit for the colder stars one can use the fact that the black body curve peaks somewhere within the observed spectra (for hotter stars the peak is shortward of 3000 Å). The toughest fit will probably be for type A2, but one can use the fact that its temperature is some-

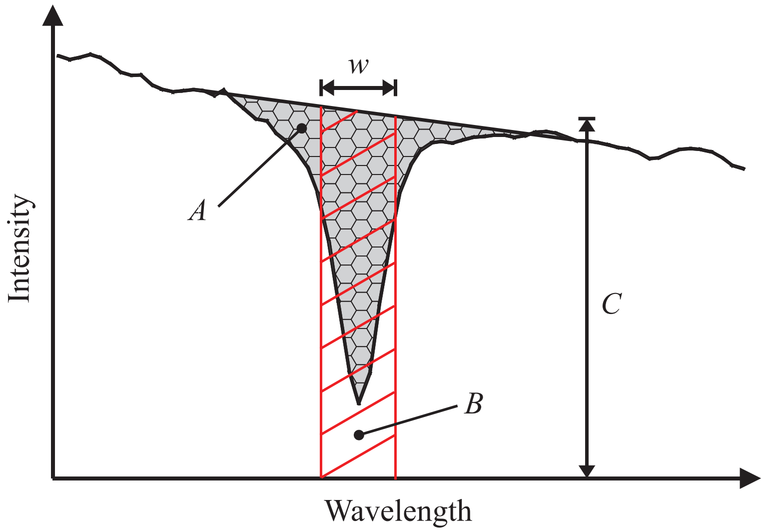


Figure 11: The equivalent width of an absorption line is defined so that the integrated grey area in the line, A, equals the red striped area, B, which is the equivalent width w times the continuum level C.

where between that found for B4 and F0. Make a list of fitted black body temperatures as a function of spectral type. Estimate the error in your fit by finding what range of temperatures still yield an acceptable fit.

Exercise 2 *Use Spectrix to investigate which five or six of the lines in Table 1 that are the strongest in the spectra of the O5, G0 and G8 stars. Try to rank these lines. You will find that there are some strong lines in the spectra that are not listed in Table 1. For this comparison however, please ignore those lines. Why are not the same lines the strongest in all stars?*

Spectral lines provide a rich source of information, and in the following exercises you will take a closer look at the Hβ absorption line to study its dependence on stellar properties.

Exercise 3 *Use the integrator in Spectrix to calculate the equivalent width $w_{\mathrm{H}\beta}$ of the Hβ line in each of the seven stars (see Fig. 11 for an example of an integration). Estimate the error you make in this measurement by noting how the result varies when you integrate the line over slightly different intervals. Note that the Hβ line will be strong in some stars, but extremely faint in others (maybe not even detected). In your report, make a plot of $w_{H\beta}$ versus the black body temperature that includes all of the seven stars you were able to measure.*

Exercise 4 *Since the Hβ absorption line arises from a transition between levels $n = 2$ and $n = 4$ in the hydrogen atom (Fig. 1), the observed equivalent width should tell us something about the number of H atoms that have an electron in level 2. In exercise 4 of the preparatory exercises, you derived n_2/n_{tot}, which is just what we need here. Assume an electron density of $n_e = 10^{21}\,m^{-3}$ and calculate this ratio for the effective temperatures you found for the seven stars (exercise 1).*

Plot in your report the Hβ equivalent width versus the fraction of hydrogen in level 2 in a logarithmic diagram. Do you see a correlation between the Hβ line strength (equivalent width) and the availability of $n = 2$ hydrogen atoms (n_2/n_{tot})?

Exercise 5 *Discuss the results obtained so far. In particular, why do the equivalent widths change with temperature the way they do in the plot? For what temperature(s) do you find the*

Element	λ [Å]	Name
Ca II	3934	K
Ca II	3968	H
H I	3970	H ϵ
Si IV	4089	
H I	4102	H δ
He II	4200	
Ca I	4227	
Fe I	4325	
H I	4340	H γ
He I	4471	
He II	4541	
O II	4650	
He II	4686	
H I	4861	H β
H I	6563	H α

Table 1: Lines to look for in the stellar atmospheres.

Element	λ [Å]	Name
N III]	1750	
C III]	1909	
[O II]	3728	
He II	4686	
H I	4861	H β
[O III]	4959	
[O III]	5007	
He I	5876	
N II]	6548	
H I	6563	H α
[N II]	6583	

Table 2: Lines to look for in the nebula. Due to the infrequent collisions between particles in nebulae, some particles reach states not attainable in stars. The emission related to those transitions are called forbidden or semi-forbidden, and are marked by [] and], respectively.

largest Hβ equivalent width? why is this? Explain also the behaviour in the equivalent width vs. n_2/n_{tot} plot. If you find the relations confusing, you may want to consult the background theory section again.

There are more parameters than the effective temperature that could affect the line strengths (equivalent widths). Two such parameters are Z (heavy mass element fraction) and log g (surface gravity) which we will explore in the following two exercises.

Exercise 6 *For spectral type G0, there are spectra included in Spectrix that cover surface gravities between* $\log g = 0.0$ *and 5.0 in steps of 1.0. Note that this range in fact covers very different gravities, since* $\log g = 0$ *means a surface gravity of only 0.01* $\mathrm{m\,s^{-2}}$ *while* $\log g = 5.0$ *corresponds to 1 000* $\mathrm{m\,s^{-2}}$ *(on Earth log g is about 3, meaning 10* $\mathrm{m\,s^{-2}}$*).*

Measure the equivalent width of the Ca H & K lines (you can integrate both at the same time since they vary the same way) for each of these spectra. Make a plot of equivalent width versus surface gravity (log g). Is there any relation? Can you think of any parameters or processes that change with surface gravity that could affect line strength?

Exercise 7 *In order to check the influence of metallicity on equivalent widths, there is one G0 spectrum with very low metallicity (*$Z = 0.002$*) and one spectrum with very high metallicity (*$Z = 0.04$*) included in Spectrix (remember that there is also the* $Z = 0.02$ *solar metallicity G0 spectrum). Use these three G0 spectra to measure the influence of heavy elements on both the Hβ and Ca H & K lines. The Ca doublet (H and K) may, as before, be integrated both at the same time. How does the equivalent width of Ca H & K change with metallicity? Why is this? What about the equivalent width of Hβ? Remember that Z is a measurement of the mass ratio of elements other than hydrogen and helium.*

5.2 Emission line spectrum

We will now study an emission line spectrum, namely the nebula surrounding the well-known supernova 1987A (see front page for images). The spectrum was obtained from the Hubble Space Telescope. One important advantage of observing from space is that light blocked by Earth's atmosphere (e.g. ultraviolet radiation) can easily be observed. This is the reason for the spectrum reaching all the way down to 1640 Å, even though ground based observatories cannot go below ~3000 Å; a limit set by the atmosphere.

The nebula is moving relative to the Earth, therefore, the spectrum gets shifted in wavelength according to the Doppler shift law for non-relativistic velocities: $\lambda_{obs} = (v_r/c + 1)\lambda_0$, where v_r is the relative radial velocity between the source and the observer, λ_0 is the (emitted) rest wavelength and λ_{obs} is the wavelength at which the line is observed.

Exercise 8 *Determine the relative radial velocity v_r between the observer and the nebula by measuring the difference between rest wavelengths and the corresponding observed wavelengths in the spectrum. For this to work we need more accurate rest wavelengths than those given in Table 2 since rounding values to the nearest wavelength, as is done when naming the lines, introduces an error much larger than in the measurements themselves. Use the following four well-determined lines: He II 4685.71, Hβ 4861.325, [O III] 5006.843, Hα 6562.80.*

Remember that according to the Doppler shift law, the doppler shift is not a constant $\Delta\lambda$, instead it will increase with wavelength. This is a good reason for not using the lines at shorter wavelengths since the measurements will not be as accurate. After you have used these lines to get four estimates of the radial velocity, calculate your best value from their mean and its errors from the standard deviation.

Exercise 9 *Identify the five strongest lines in the spectra, but only use lines listed in Table 2. To keep things from getting too time consuming, it is enough to use the peak fluxes of the lines (on top of the continuum) for estimating their relative strengths. Again, there are many lines in the spectrum that are not listed in the Table, but concentrate on those lines that are. Lines marked with] or [ ] are called semi-forbidden and forbidden, respectively. They are only seen in low-density gas of nebulae, and never in stars or on the Earth (this is in fact the historical reason for their name). This is the case since there are frequent collisions between atoms in high-density environments, effectively replacing these very rare transitions (that need atoms to be undisturbed for a very long time) with collisional transitions. Take care to properly compensate for the Doppler shift when you identify a line that is close to others, as there is an obvious risk to choose a bluer line, if the Doppler shift is neglected.*

In the following exercises you will determine the abundance of helium (relative to hydrogen) by measuring line strengths. This can be done since the strength of a line is proportional to the number of elements, as long as the mean free path of a photon in the line is greater than the size of the radiating medium (i.e. the line is not optically thick).

Exercise 10 *Begin by measuring the flux in the Hβ line of hydrogen, the 5876 Å line of He I and the 4686 Å line of He II. Be careful to identify the lines correctly. Vary the integration intervals slightly to get a feeling for how sensitive the integrals are to the choice of interval, and use this to estimate the errors of your measurements. The units of these flux measurements are arbitrary, however, this does not matter since we will be using flux ratios in the next exercise.*

The hydrogen and helium lines are *recombination lines* that arise when an electron is captured by an ion (when it "recombines"). The electron is then often captured to an excited state, that is, an energy level above the ground level, and subsequently transits down to levels of lower energy and ultimately to the ground level. During such transitions photons are emitted, and observed later as recombination lines in the spectrum.

The intensity of such a line can be written $F_{\rm line} \propto C_{\rm line} n_{\rm ion} n_{\rm e}$, where $C_{\rm line}$ is a constant, quantifying how much energy is emitted by the particular line on average, for each recombination.

Since these lines are recombination lines, the Hβ line measures the amount of H II, 5876 Å He I the amount of He II and 4686 Å He II the amount of He III. The measurements are thus not sensitive to the amount of neutral hydrogen and helium, but since the nebula is highly ionised, we may assume that neutral atoms are negligible in comparison to ionised atoms. Using this approximation we can rewrite the number densities of hydrogen and helium:

$$n_{\rm H} = n_{\rm H\,I} + n_{\rm H\,II} \approx n_{\rm H\,II}$$
$$n_{\rm He} = n_{\rm He\,I} + n_{\rm He\,II} + n_{\rm He\,III} \approx n_{\rm He\,II} + n_{\rm He\,III}$$

Exercise 11 *Use the relative line strengths you measured in exercise 10 and the approximations given above to calculate* $n_{\rm He}/n_{\rm H}$*. Note that there is no need to assume a specific electron number density* n_e *since this parameter will cancel out in the ratio. Use the following set of recombination constants:*

$$C_{\lambda 5876}/C_{\rm H\beta} = 1.35$$
$$C_{\lambda 4686}/C_{\rm H\beta} = 12.0$$

Estimate the error of your derived ratio. Compare this ratio with the "normal" cosmic ratio of $n_{\rm He}/n_{\rm H} \approx 0.09$*. Does your ratio differ significantly from the cosmic ratio, and in that case, do you have any suggestion as to why?*